Types as Weak ω-Groupoids

Brandon Shapiro

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School and Workshop on Univalent Mathematics

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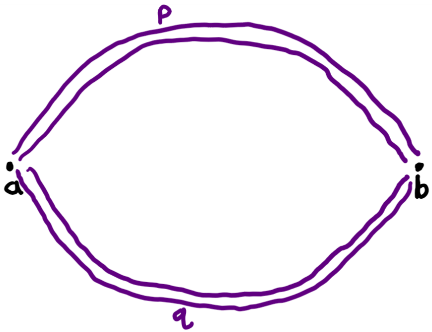

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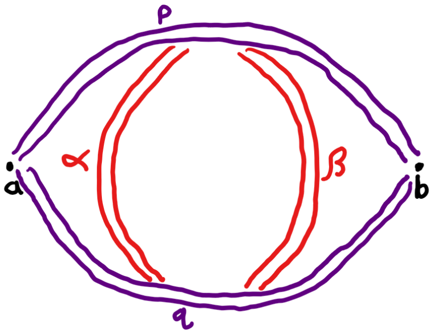

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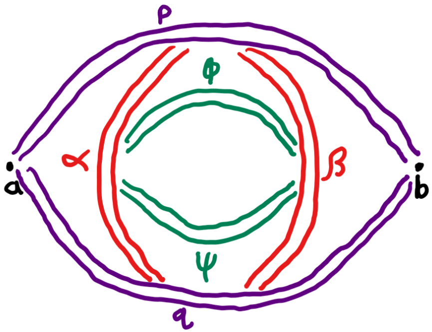

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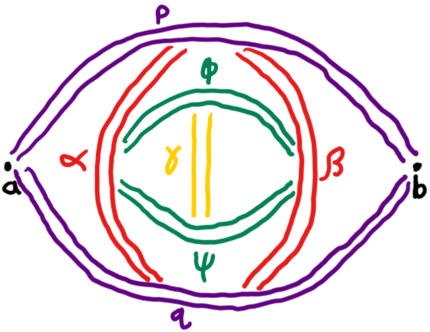

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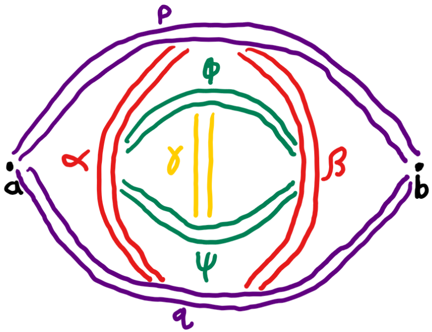

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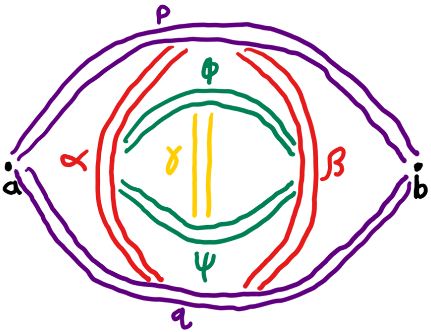

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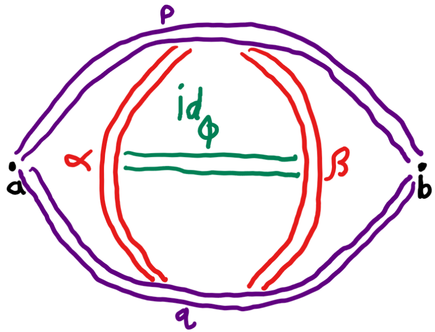

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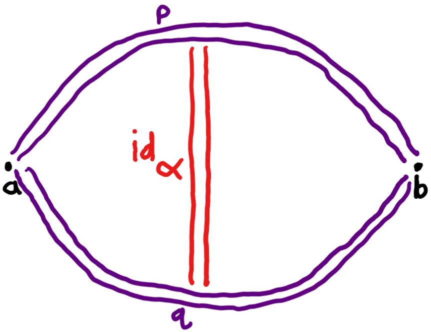

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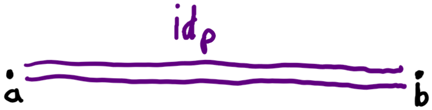

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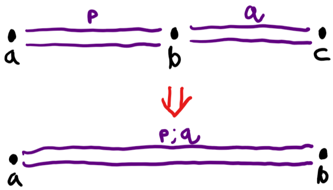

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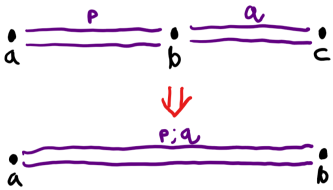

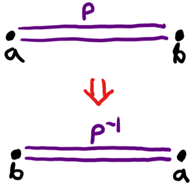

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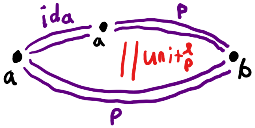
- Units.



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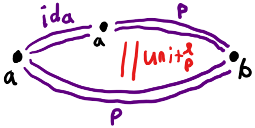

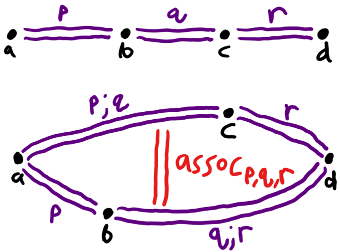

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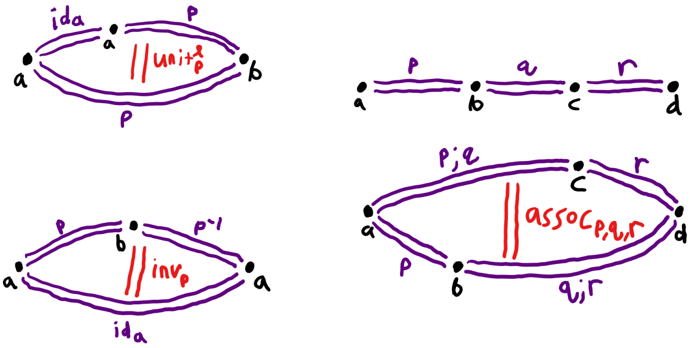

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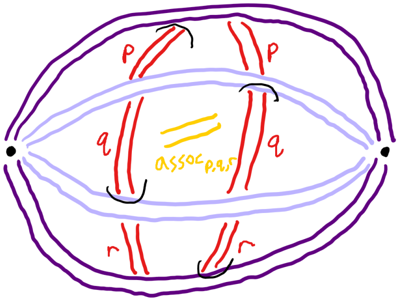

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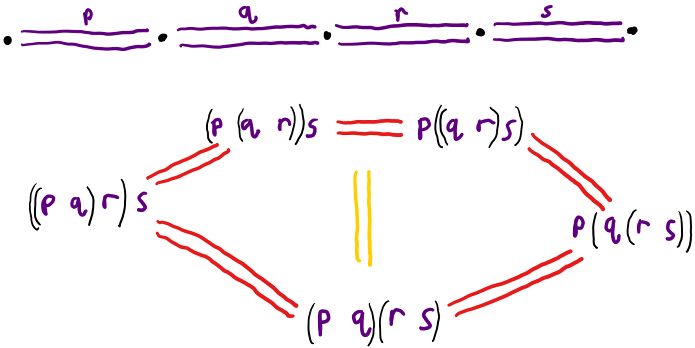

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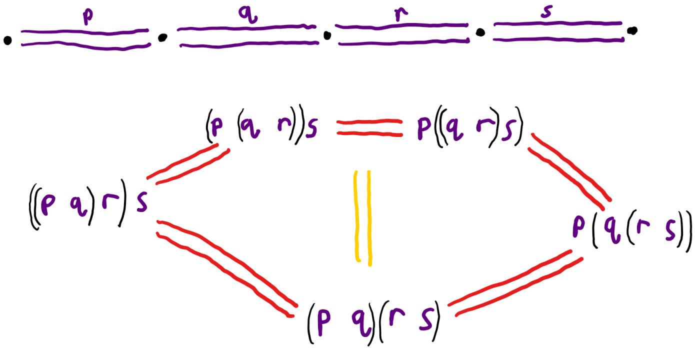
- At every level, but only up to higher cells.
- Higher order properties... What is this structure?



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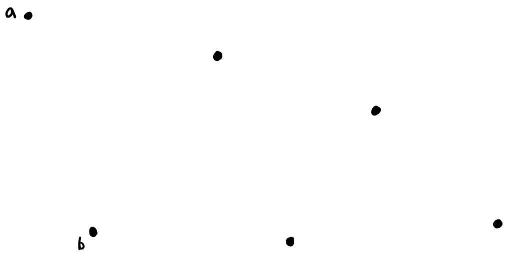

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- Sets have objects in X_0 (like $a, b : A$)
- Graphs are sets with arrows in X_1 (like $\phi : a =_A b$)

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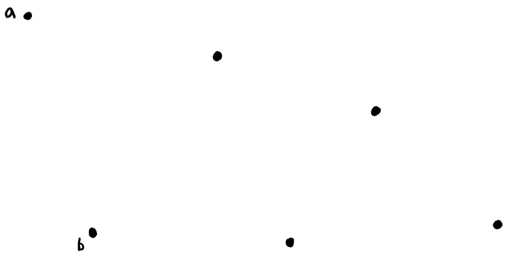

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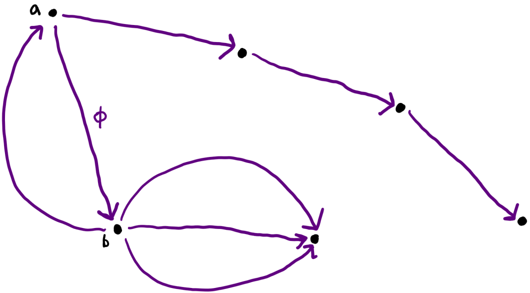

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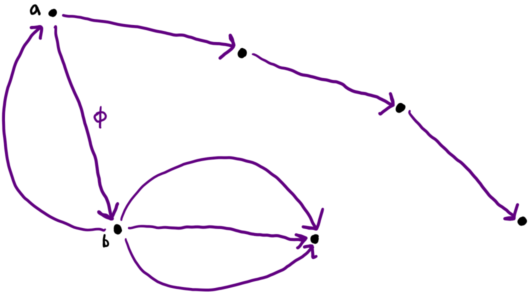

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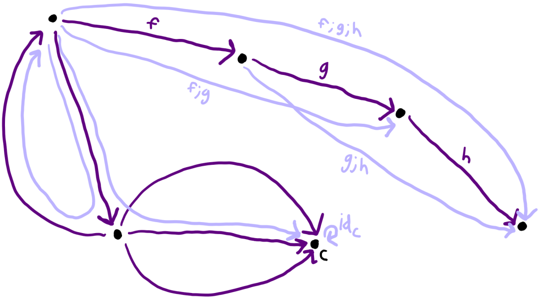

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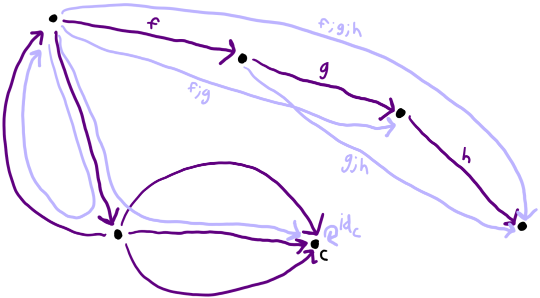

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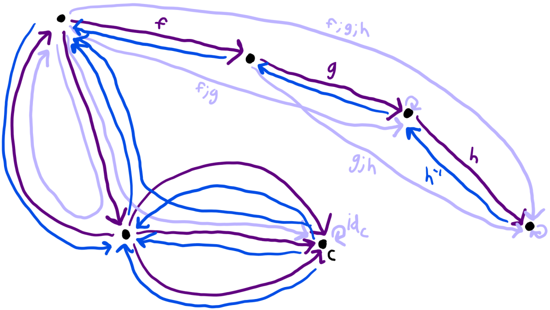

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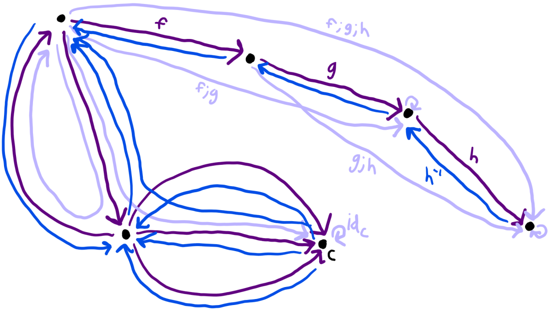

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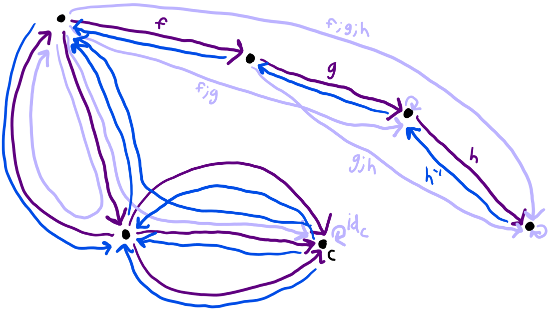

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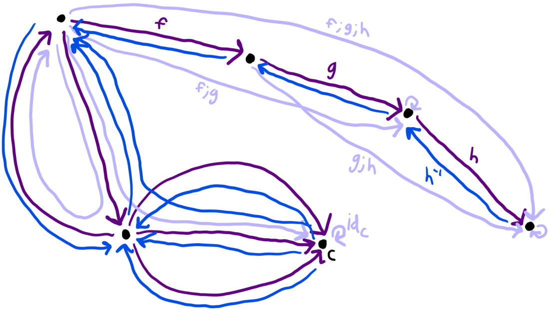

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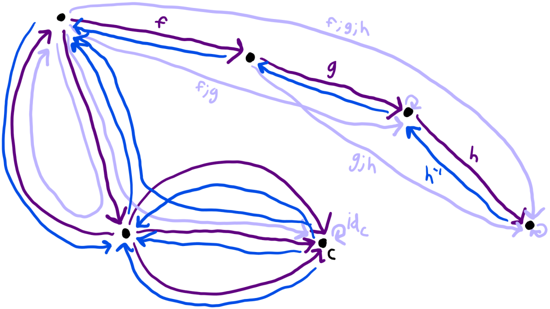

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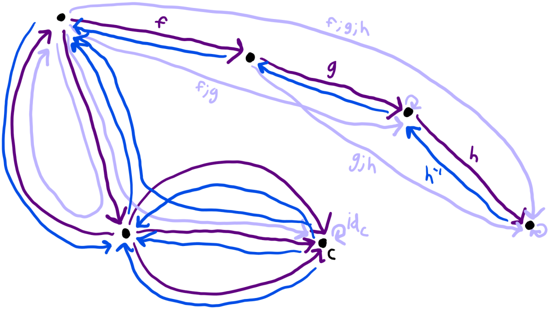

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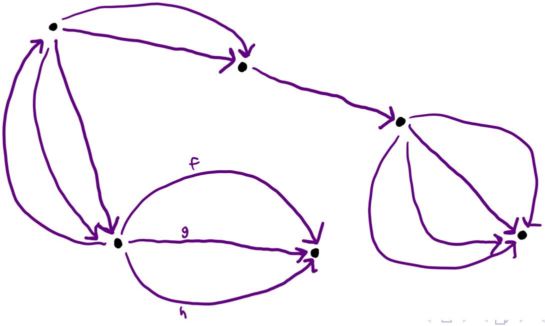

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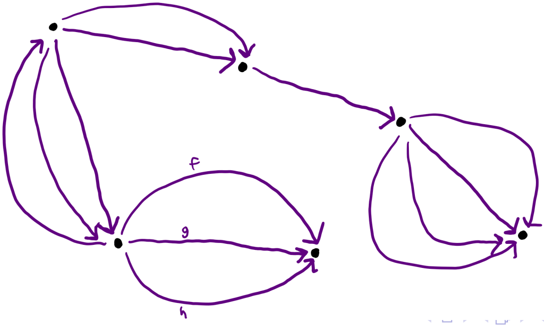

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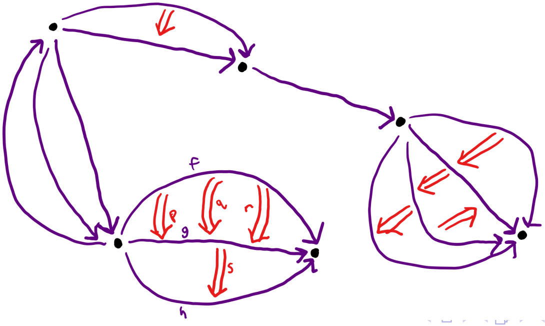

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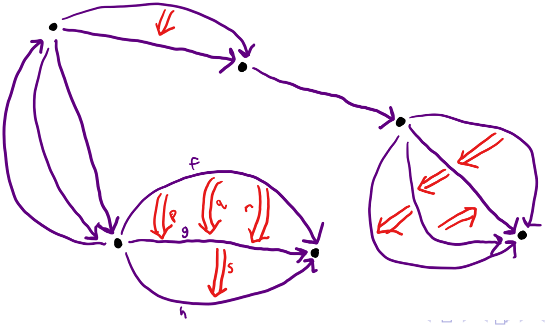

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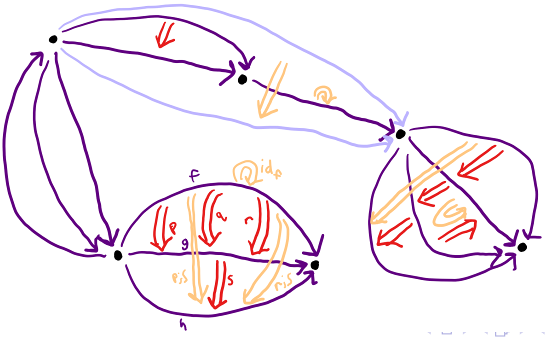

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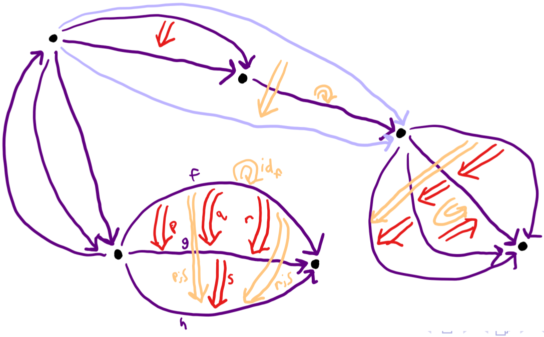

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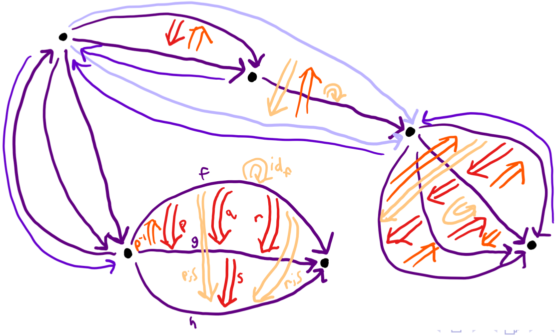

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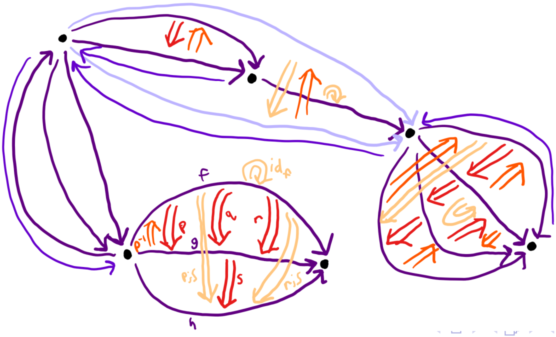

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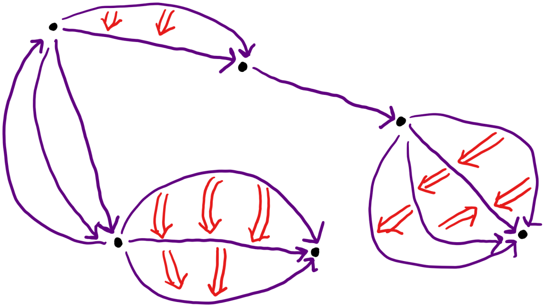

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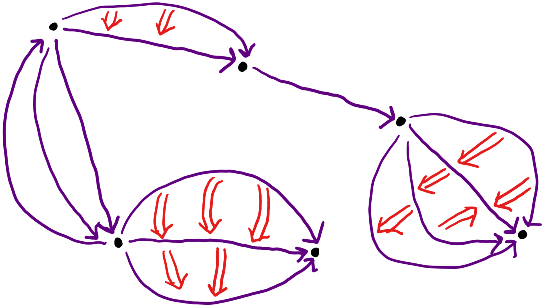

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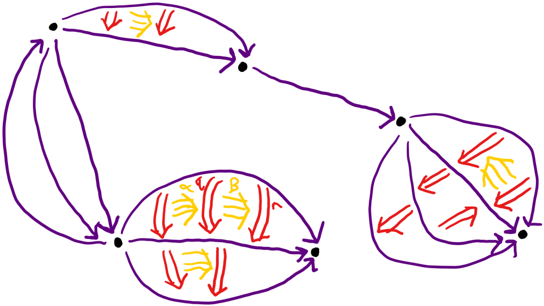

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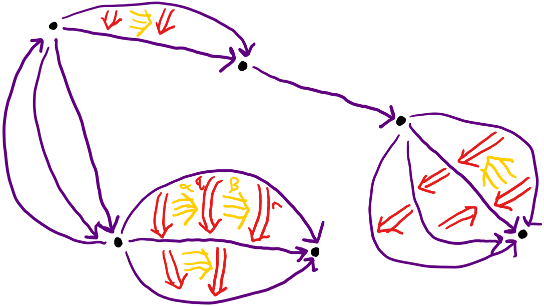
- 3-Graphs are 2-graphs with arrows in X_3
- 3-Categories are 3-graphs with composition, units, associativity
- 2-Groupoids are 2-categories with inverses

$$X_0 \underset{t}{\overset{s}{\leftleftarrows}} X_1 \underset{t}{\overset{s}{\leftleftarrows}} X_2 \underset{t}{\overset{s}{\leftleftarrows}} X_3$$



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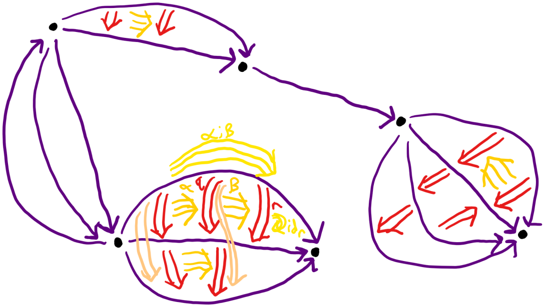

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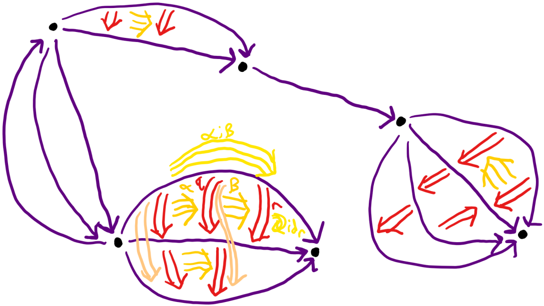

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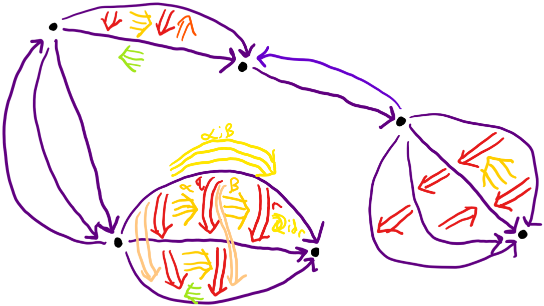

- *n*-Graphs have $X_0, \cdots, X_{n-1}$, arrows in X_n
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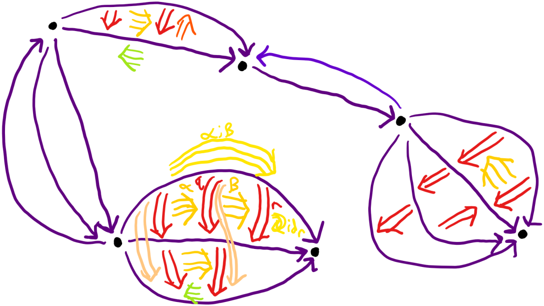

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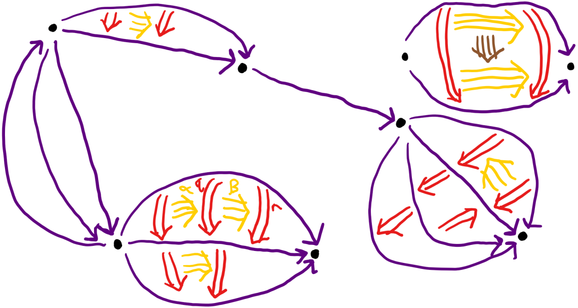

Composition Structures

- n-Graphs have $X_0, \cdots, X_{n-1}$, arrows in X_n
- $(n+1)$-Graphs are n-graphs with arrows in X_{n+1}
- 3-Categories are 3-graphs with composition, units, associativity
- 3-Groupoids are 3-categories with inverses

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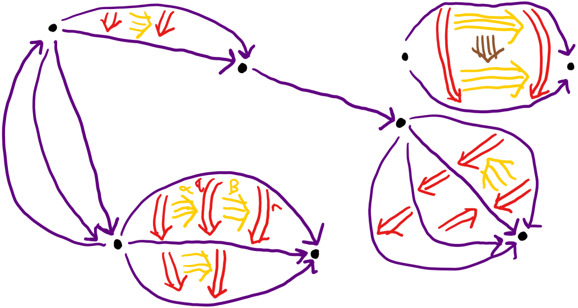

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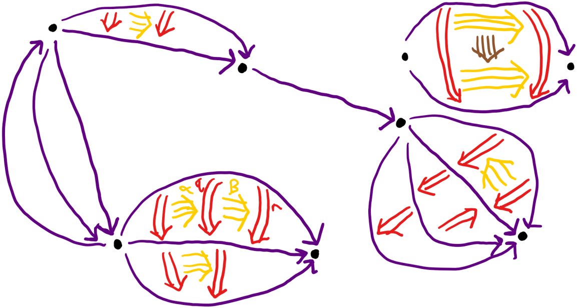

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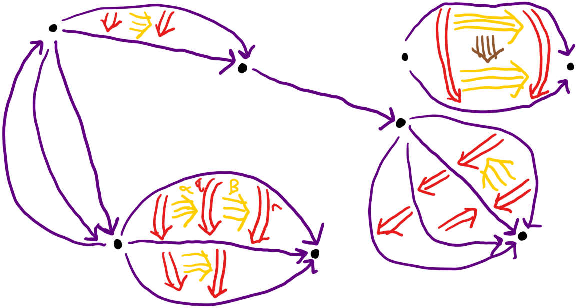

Composition Structures

- ω-Graphs have X_0, X_1, $X_2, \cdots$
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- n-Categories are n-graphs with composition, units, associativity
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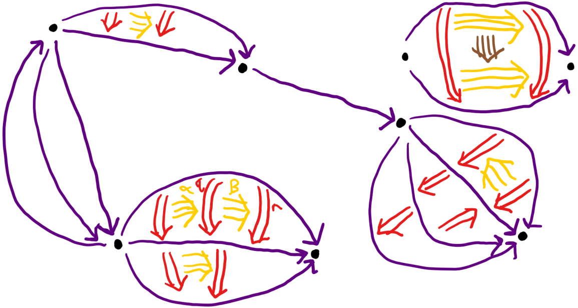

Composition Structures

- ω-Graphs have X_0, X_1, X_2, $\cdots$
- ω-Graphs are called globular sets, arrows in X_n are n-cells
- n-Categories are n-graphs with composition, units, associativity
- n-Groupoids are n-categories with inverses

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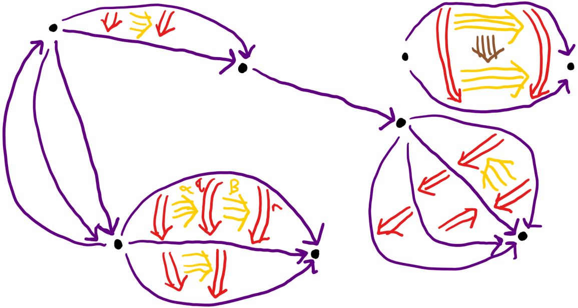

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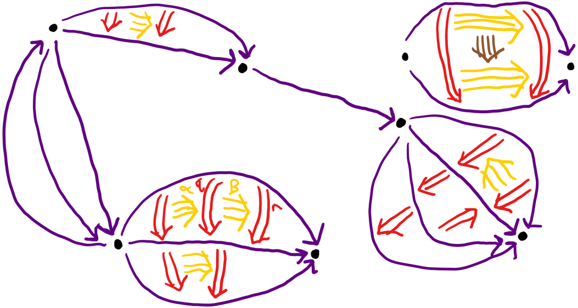

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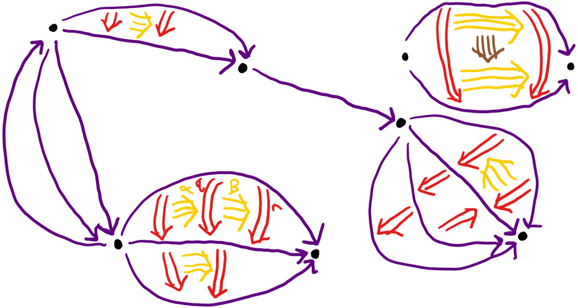

Composable Shapes

Composable Shapes

- Let X be a globular set

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- What should we be able to compose, and into what?

Composable Shapes

- Let X be a globular set

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- What should we be able to compose, and into what?
- Free unbiased pasting diagrams

Composable Shapes

- Let X be a globular set

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- What should we be able to compose, and into what?
- Free unbiased pasting diagrams in dimension 0

Composable Shapes

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- What should we be able to compose, and into what?
- Free unbiased pasting diagrams in dimension 0

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- What should we be able to compose, and into what?
- Free unbiased pasting diagrams in dimension 0

Composable Shapes

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- What should we be able to compose, and into what?
- Free unbiased pasting diagrams in dimension 1

Composable Shapes

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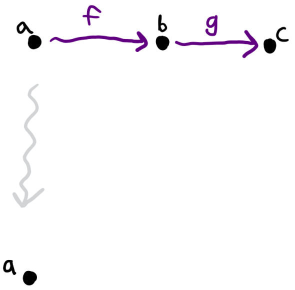

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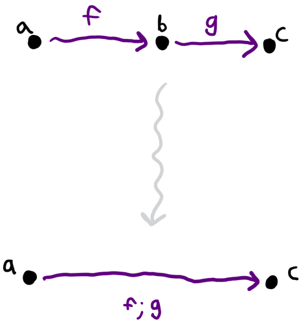

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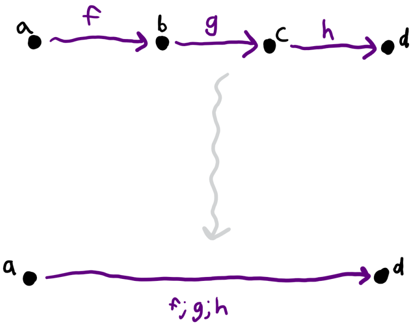

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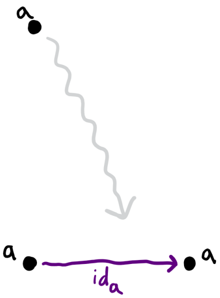

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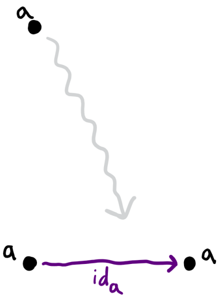

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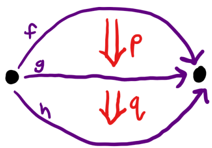

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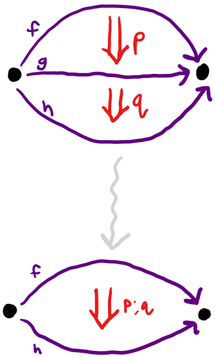

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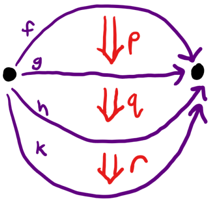

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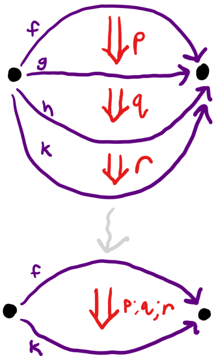

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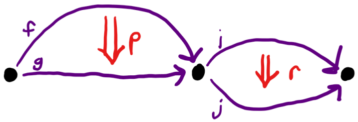

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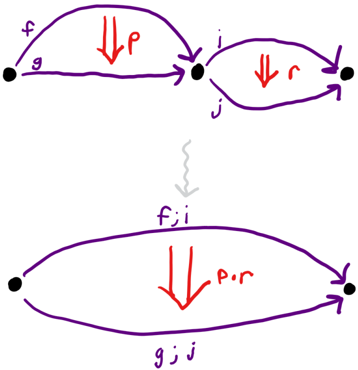

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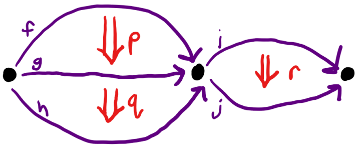

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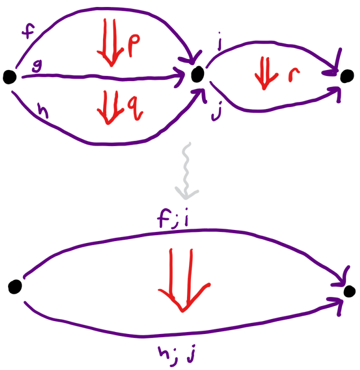

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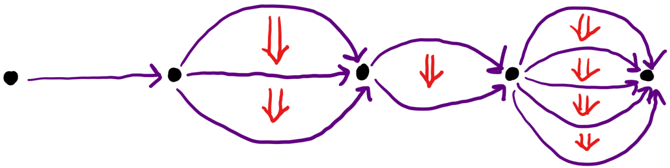

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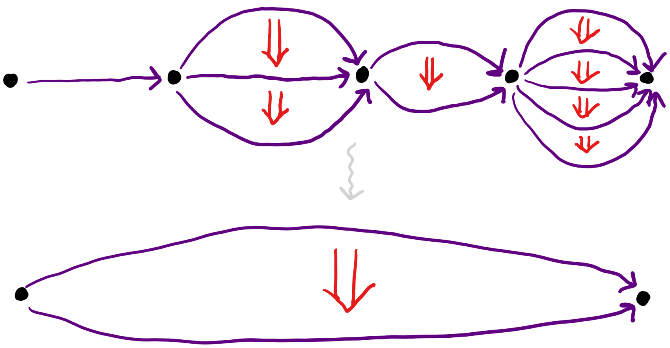

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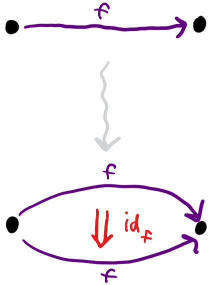

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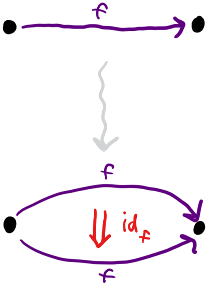
- What should we be able to compose, and into what?
- Free unbiased pasting diagrams in dimension 3



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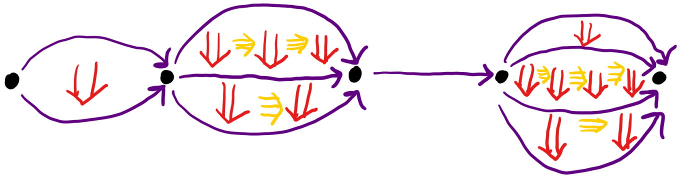

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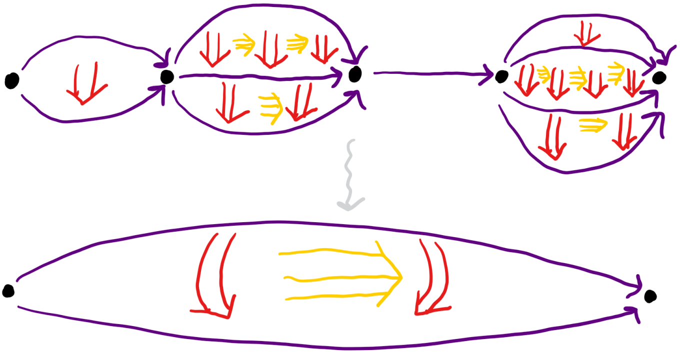

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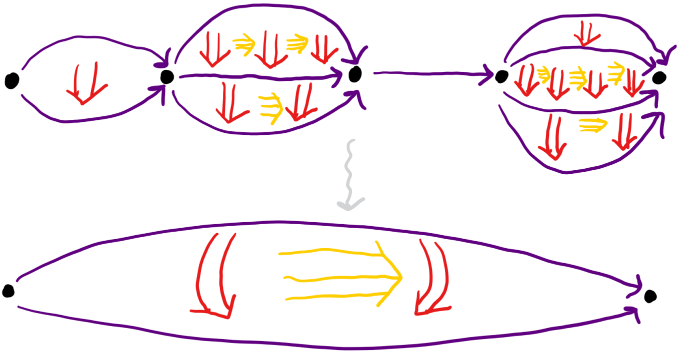

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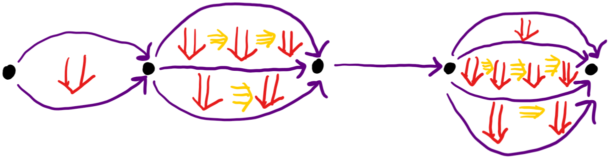

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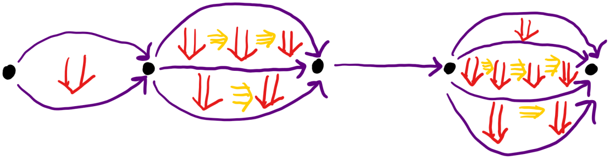

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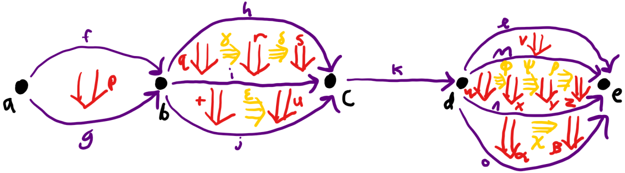

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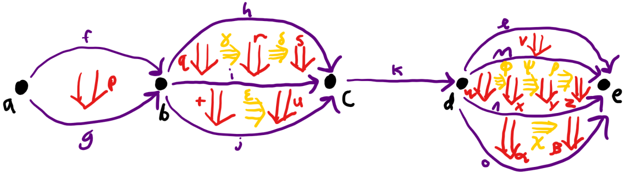

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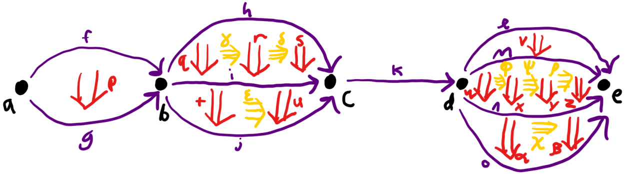

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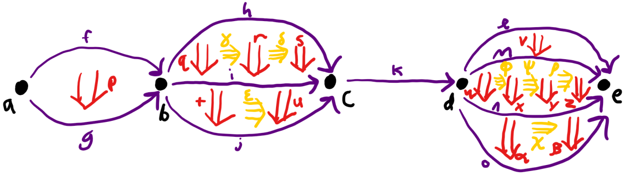

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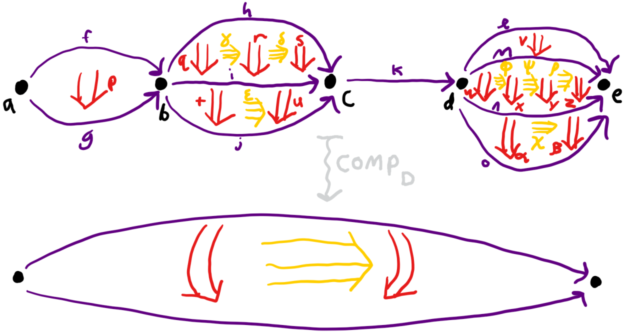

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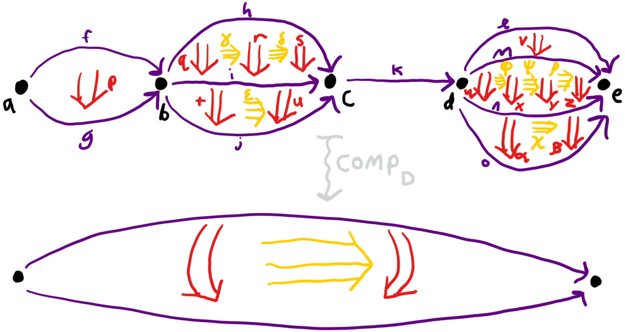

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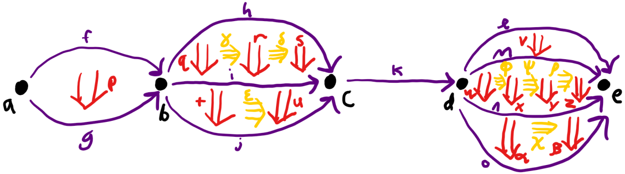

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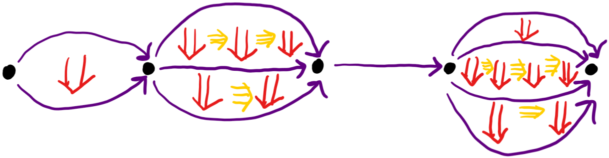

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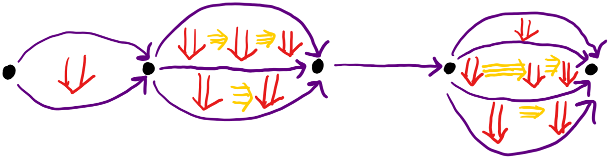

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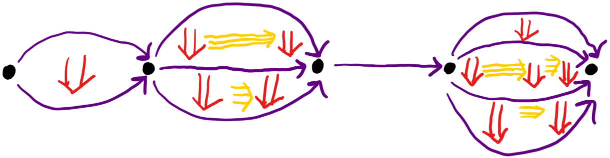

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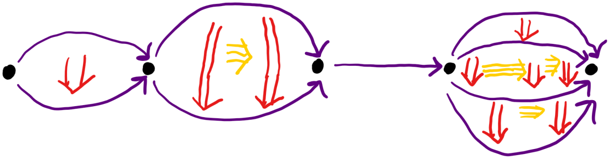

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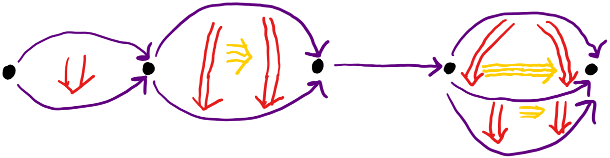

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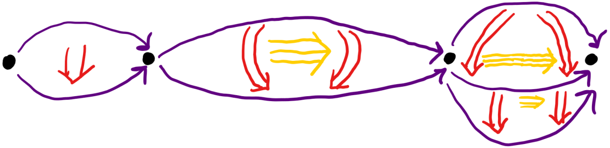

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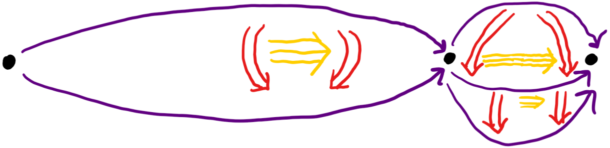

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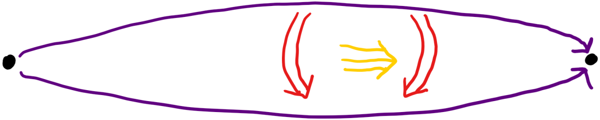

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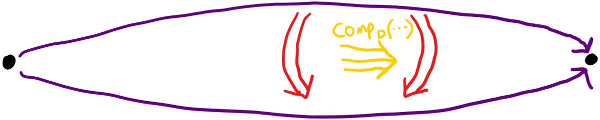

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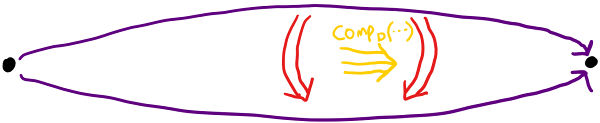

All composition orders give the same result

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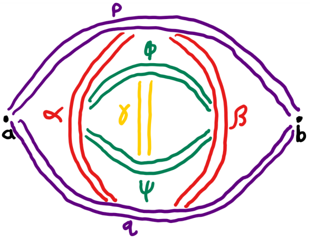

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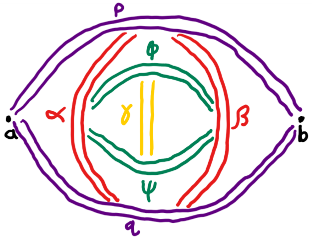

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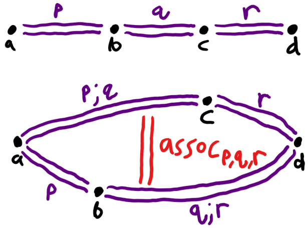

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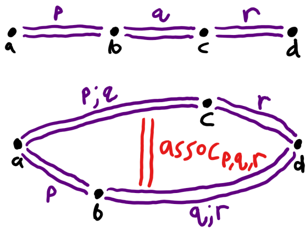

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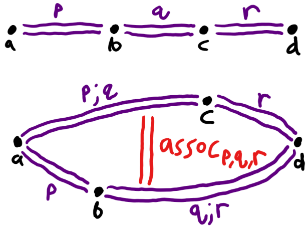
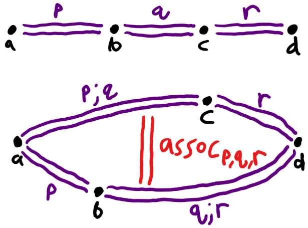


Globular Operads



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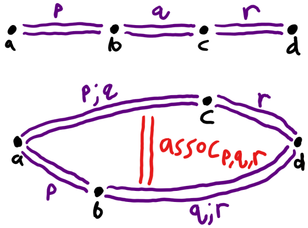

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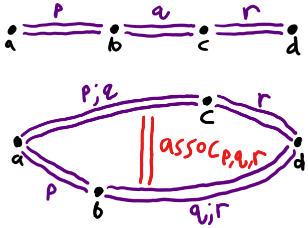

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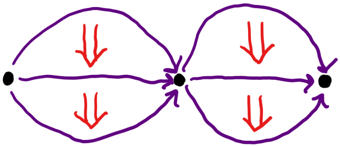

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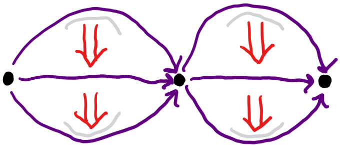

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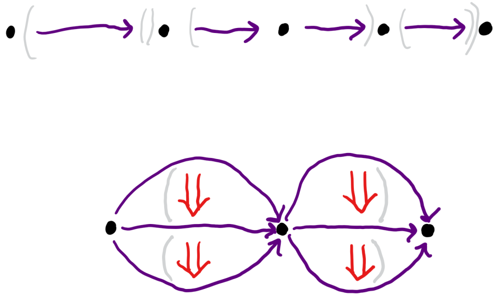

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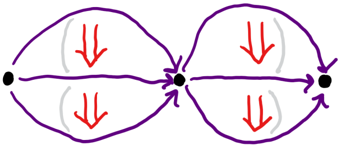

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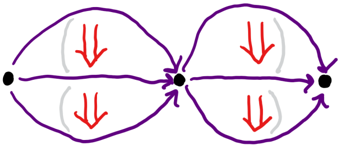

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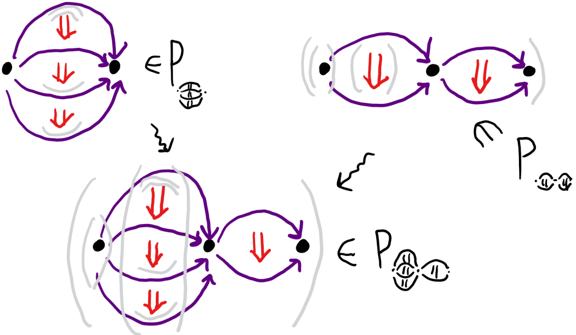

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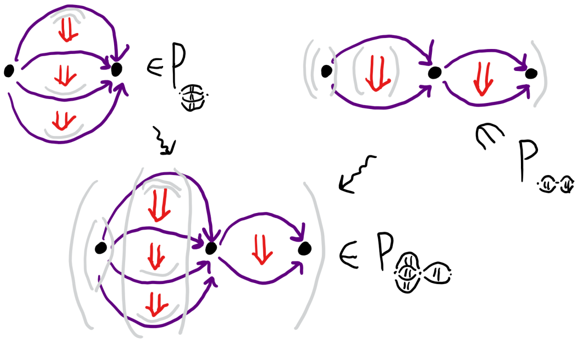

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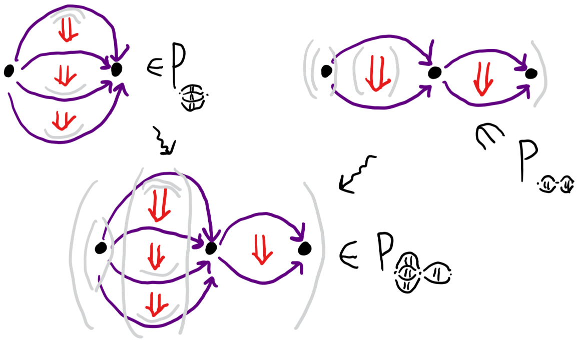

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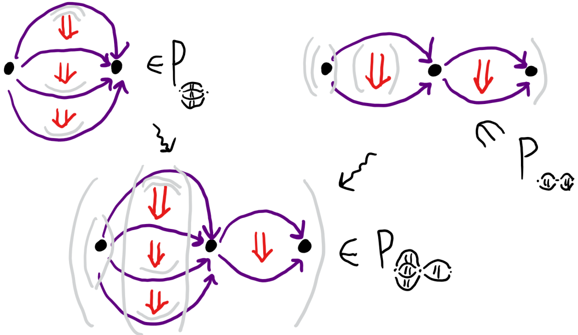

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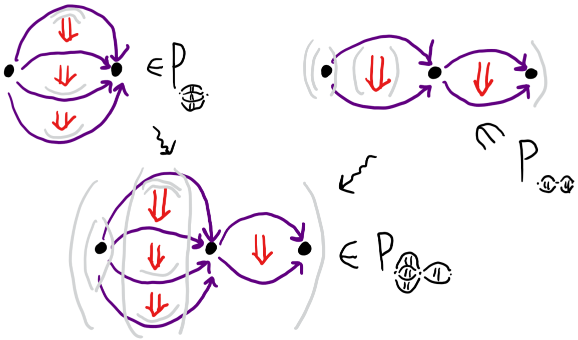

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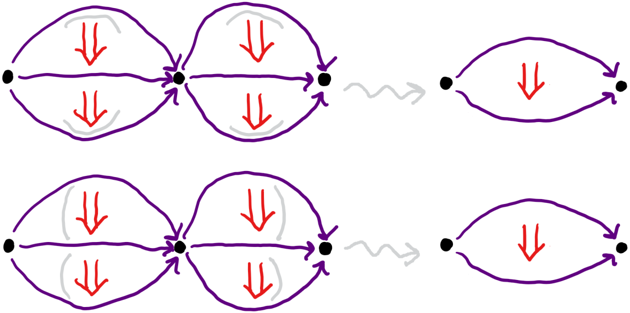

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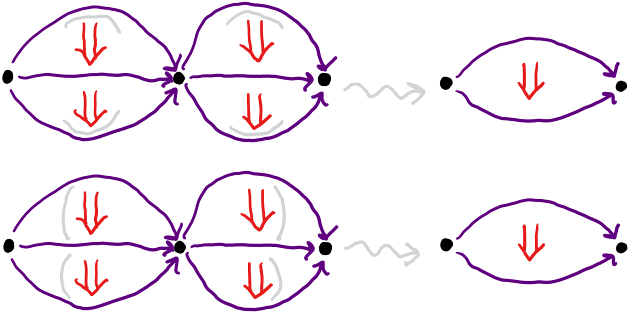

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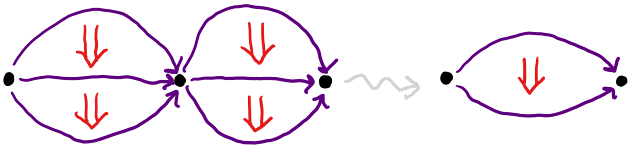

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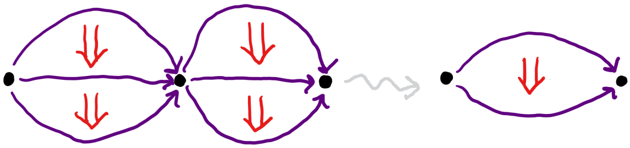

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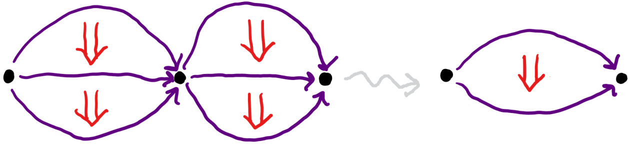

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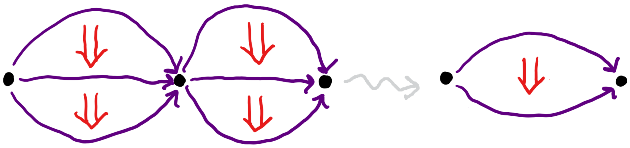

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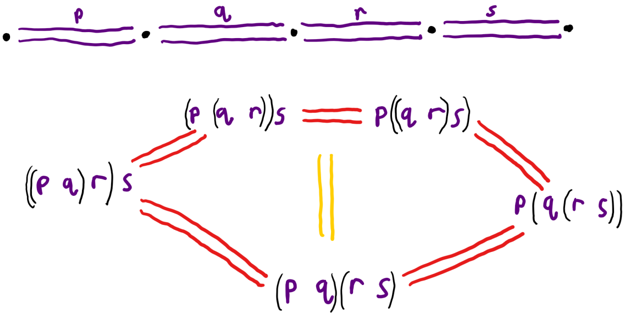

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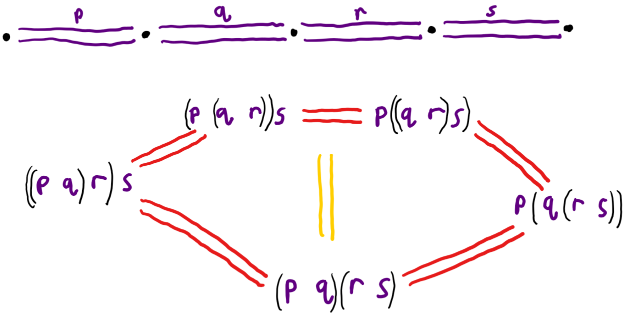

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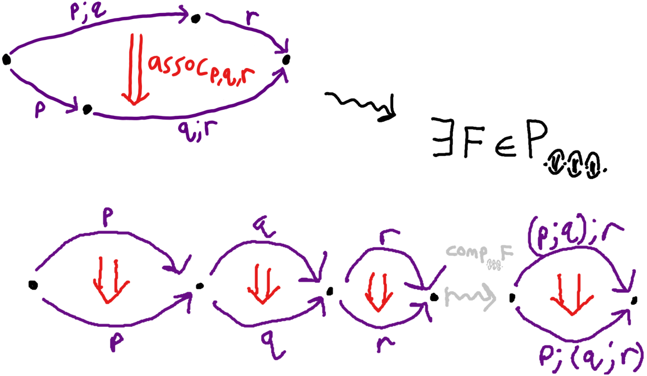

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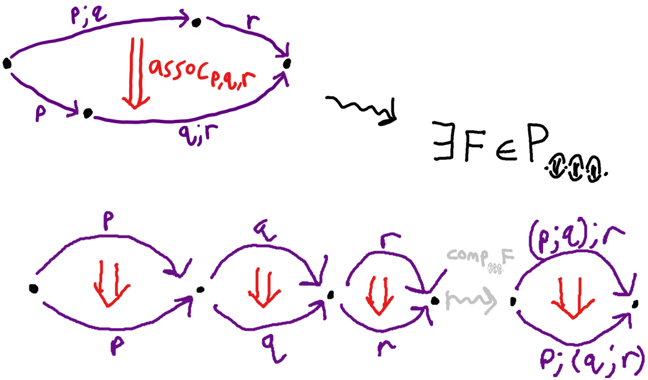

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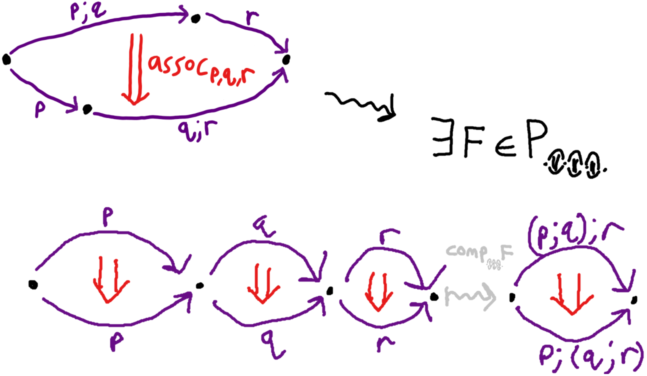

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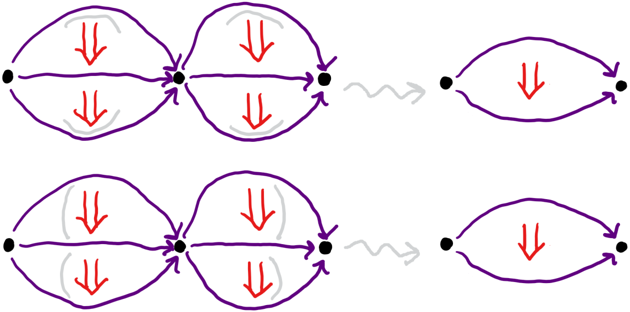

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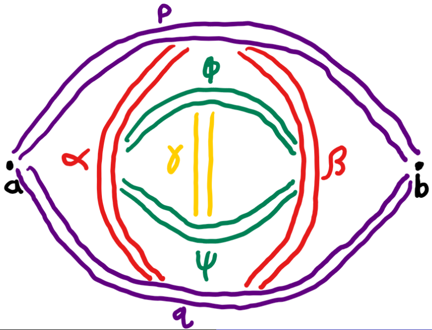

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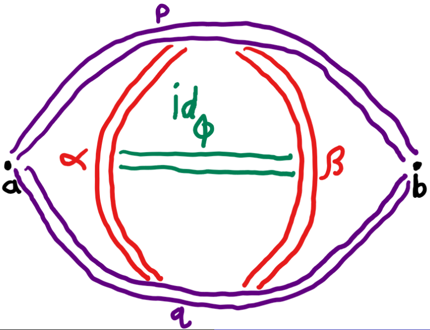

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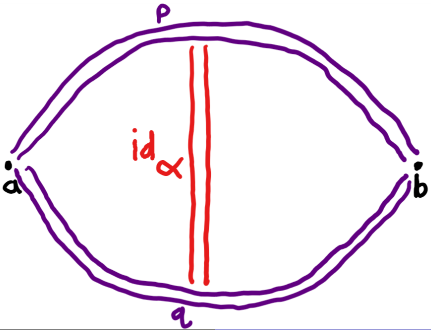

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- Is that all???

Fin

Thank you!

References

- Weak ω-groupoids in type theory:
  Benno van den Berg, Richard Garner. Types are Weak ω-Groupoids.
- More definitions of higher categories:
  Tom Leinster. Higher Operads, Higher Categories.