Shape Independent Category Theory

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Category Theory OctoberFest 2019

Categories with Different Cell Shapes

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- Categories
  - dots, arrows

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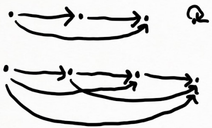

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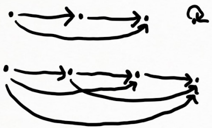
- 2-Categories – dots, arrows, 2-globs



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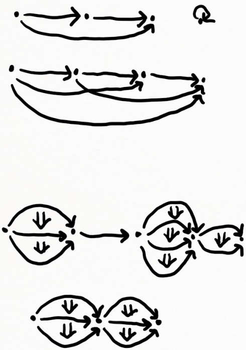

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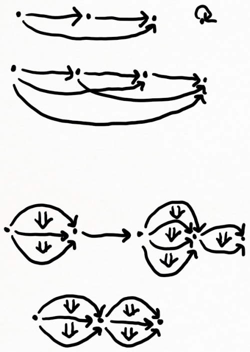

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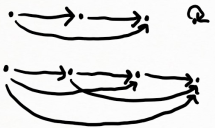

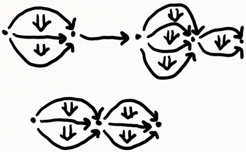

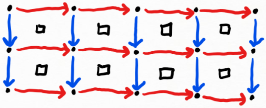

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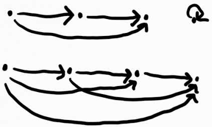

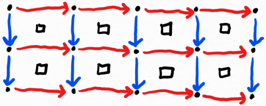

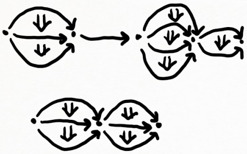

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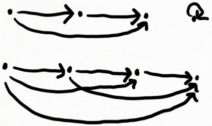

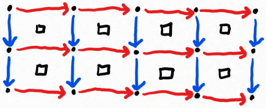

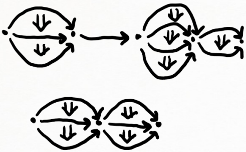

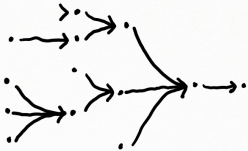

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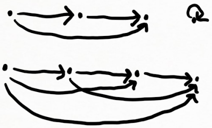

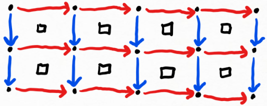

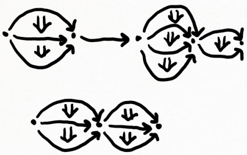

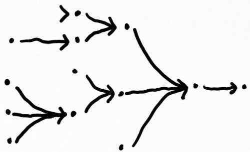

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- Categories $\to \widehat{\Delta}$ simplicial sets
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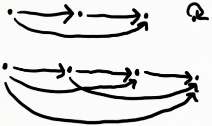

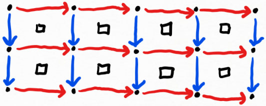

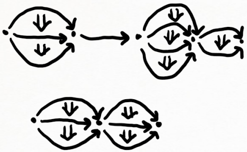

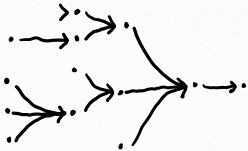

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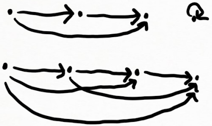

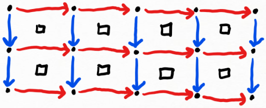

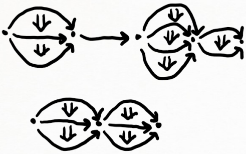

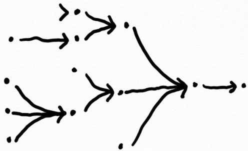

Nerves

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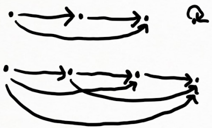
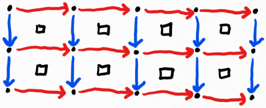
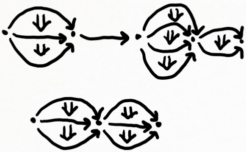
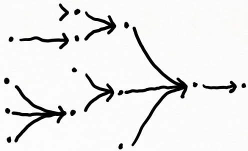
• Double-Categories	$\to \widehat{\Delta \times \Delta}$	bisimplicial sets
• Multi-Categories		









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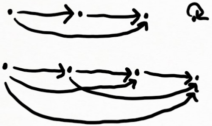
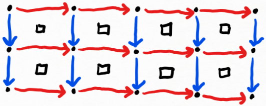
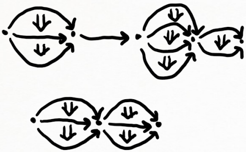
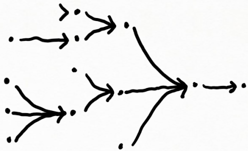
- Multi-Categories $\rightarrow \widehat{\Omega}$ dendroidal sets









Familial Monads on Cell Diagrams

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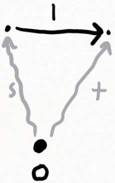

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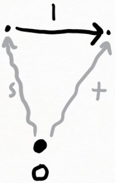

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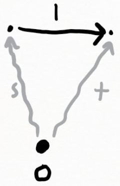

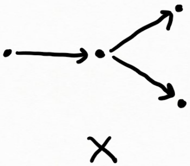

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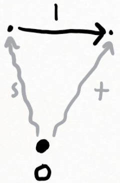

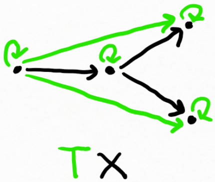

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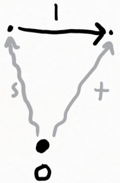

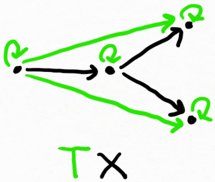

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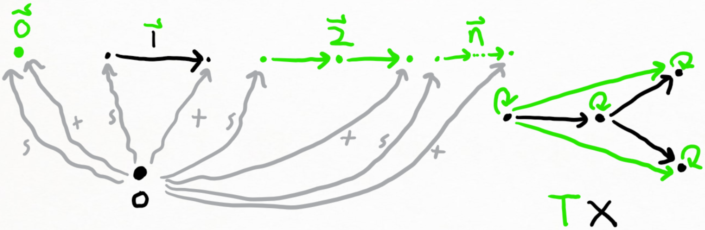

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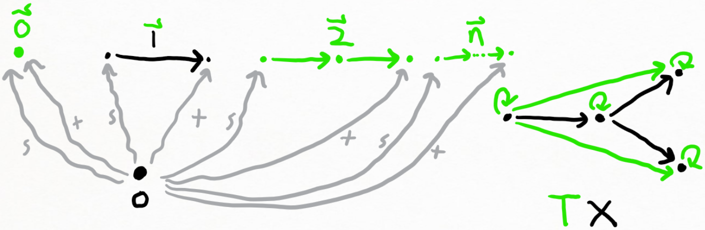

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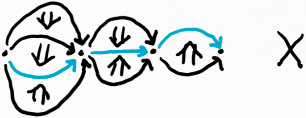

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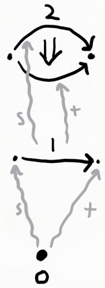

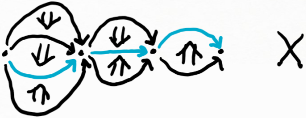

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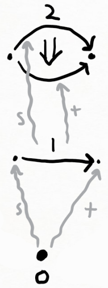

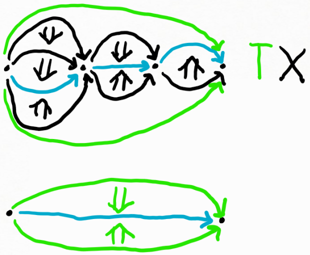

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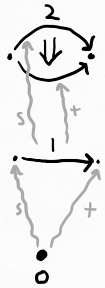

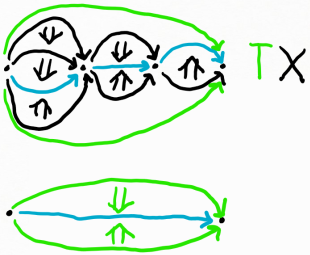

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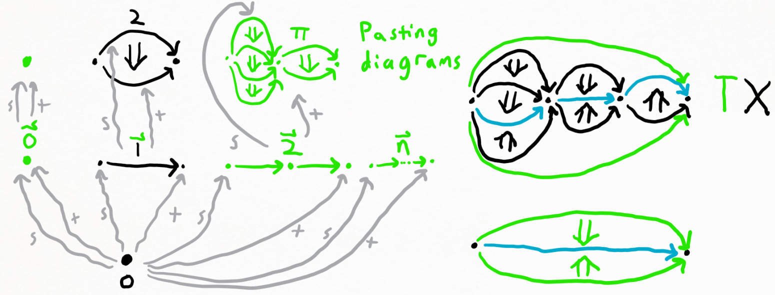

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- $G_1 \times G_1$ is the category $\begin{array}{ccc} 0 & \rightrightarrows & \color{blue}{1_v} \\ \downdownarrows & & \downdownarrows \\ \color{red}{1_h} & \rightrightarrows & 2 \end{array}$

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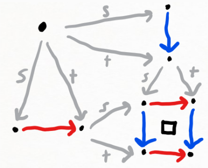

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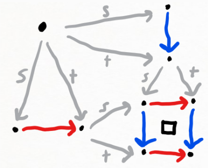

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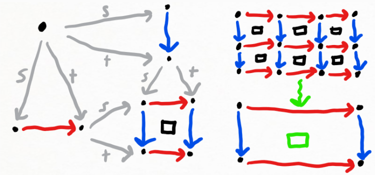

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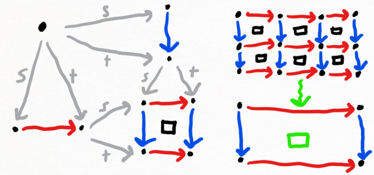

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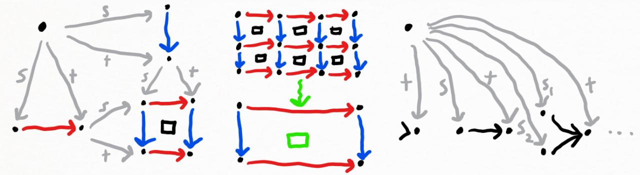

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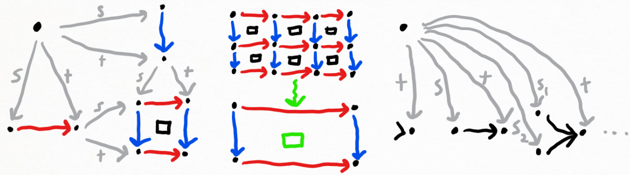

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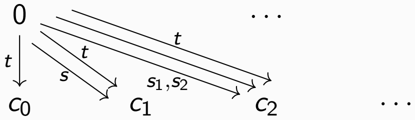

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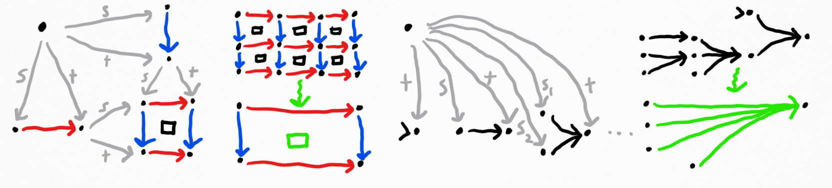

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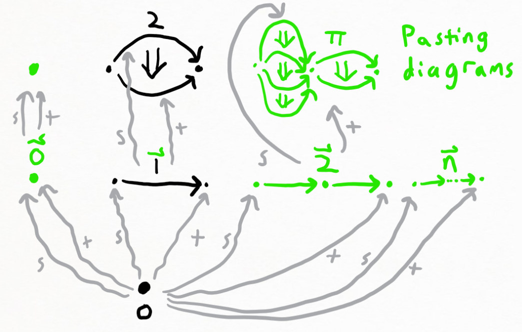
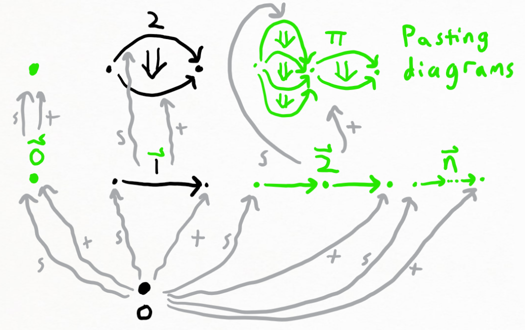


Familial Monads on Cell Diagrams

- The data of a familial endofunctor F on $\hat{\mathcal{C}}$ consists of:



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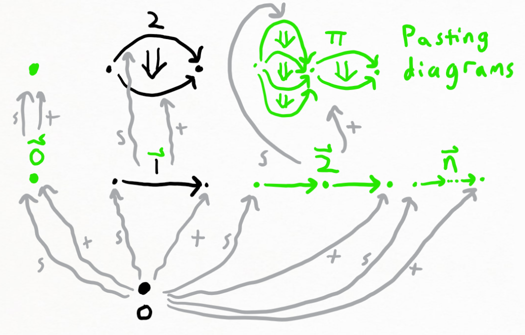

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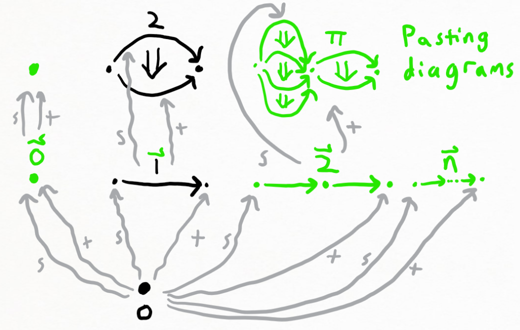

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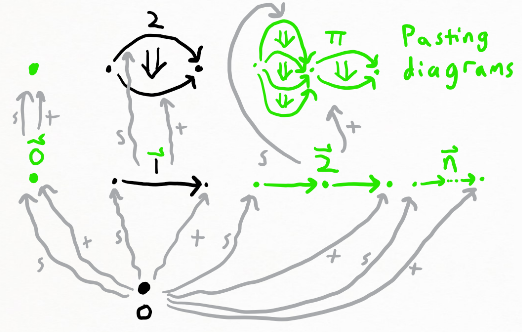
- For c in $\mathcal{C}$, X in $\hat{\mathcal{C}}$, $FX_c = \coprod_{t \in Sc} Hom_{\hat{\mathcal{C}}}(Et, X)$



Theories and Nerves

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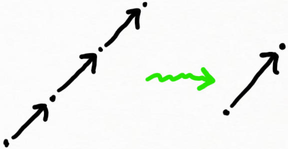

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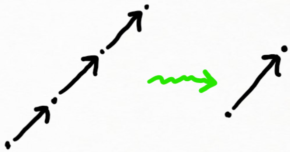

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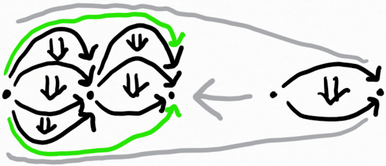

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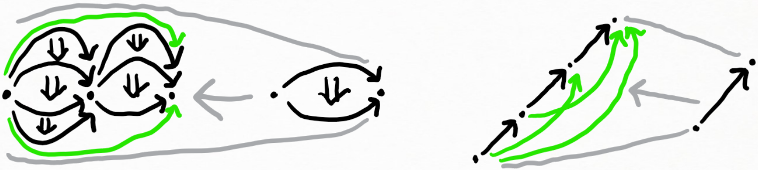

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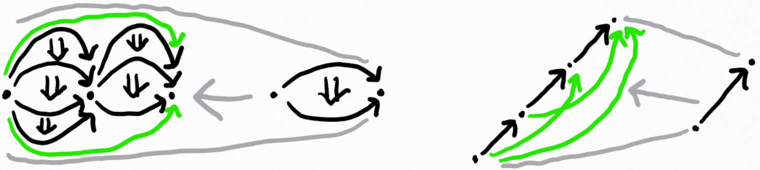

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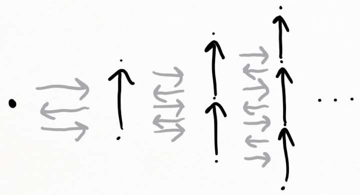

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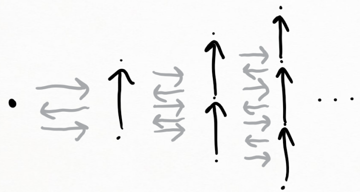

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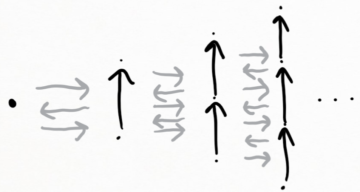

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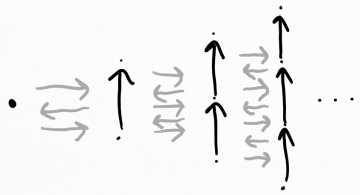
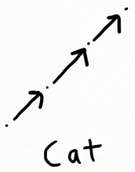
- Those are all test categories...



Shape Independent Category Theory

- Ideas from category theory should generalize to other familial algebras in cell diagrams (and often do!)



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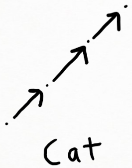

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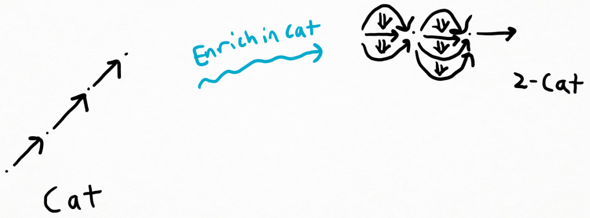

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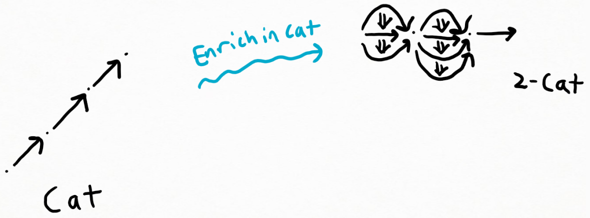

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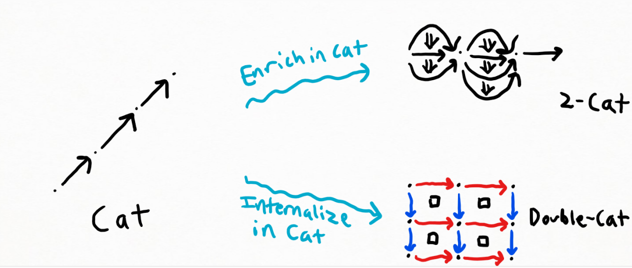

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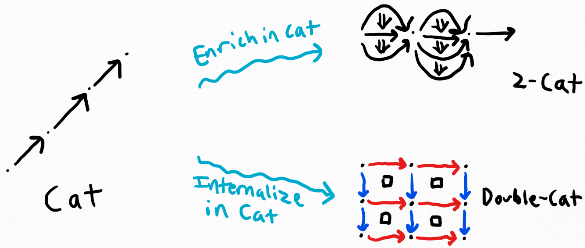
- These constructions extend to other familial representations



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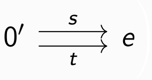

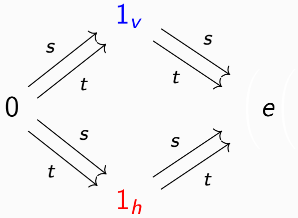

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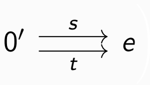

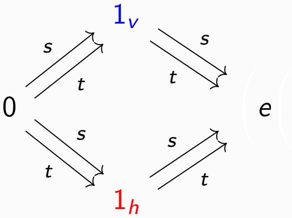

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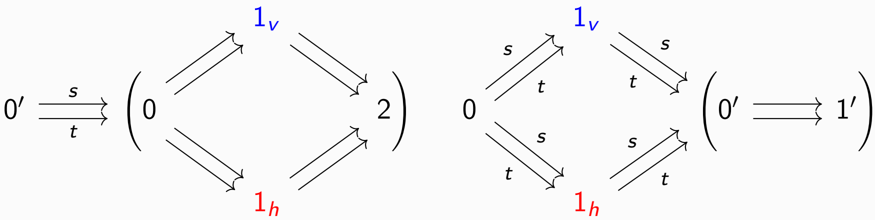

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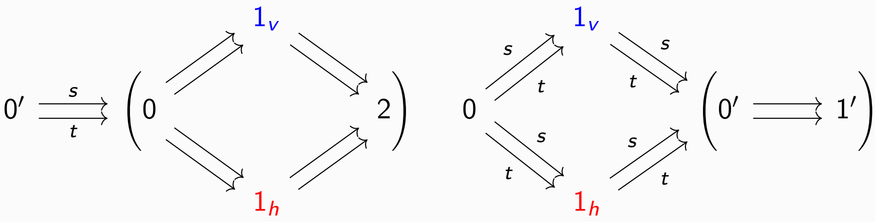

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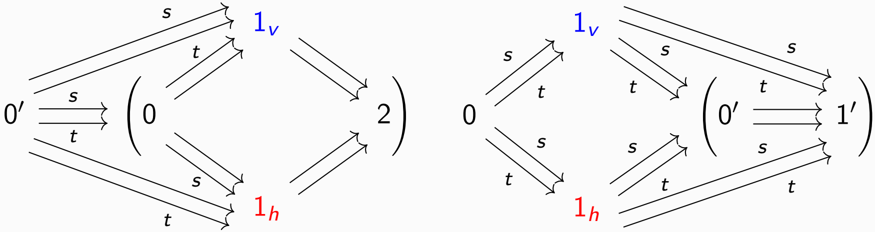

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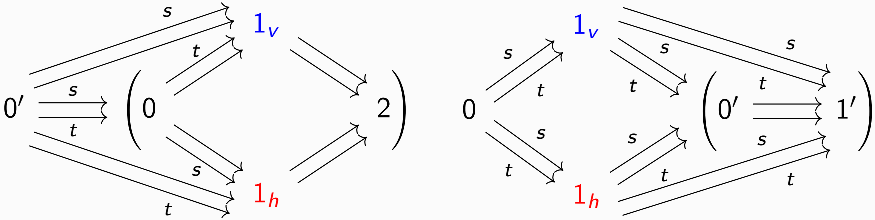

Enrichment via Cell Shapes

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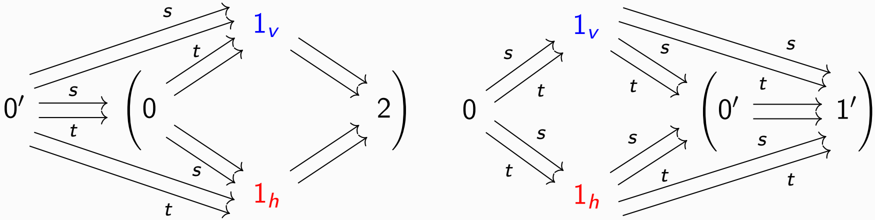

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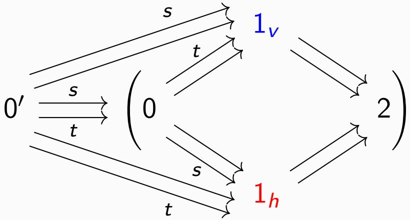

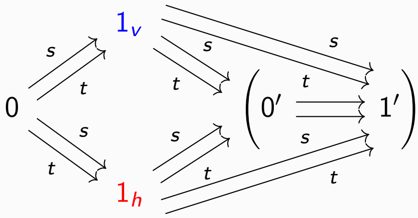

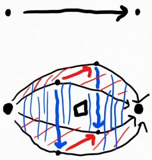

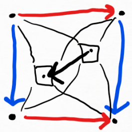

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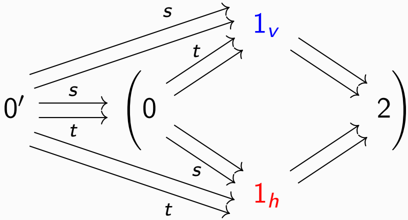
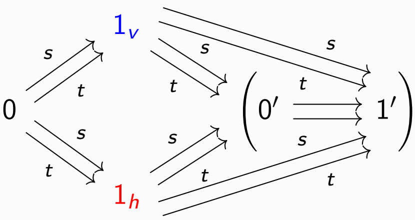
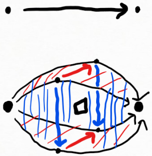
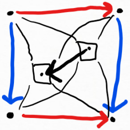
- Cell diagrams in $\widehat{\mathcal{C} \wr_e \mathcal{D}}$ are cell diagrams over $\mathcal{C}$ stuffed with a diagram over $\mathcal{D}$ in each e-cell



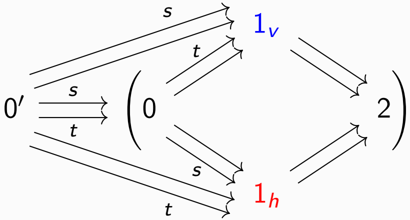


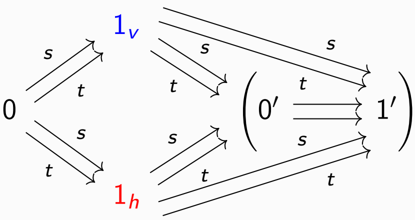


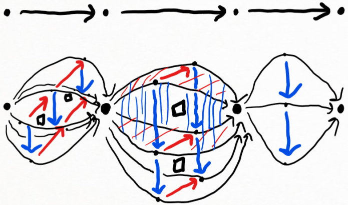


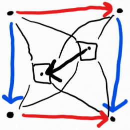
Enrichment via Cell Shapes

- Cell shapes of $\mathcal{C} \wr_e \mathcal{D}$ are e-cells stuffed with cell shapes of $\mathcal{D}$
- Cell diagrams in $\widehat{\mathcal{C} \wr_e \mathcal{D}}$ are cell diagrams over $\mathcal{C}$ stuffed with a diagram over $\mathcal{D}$ in each e-cell

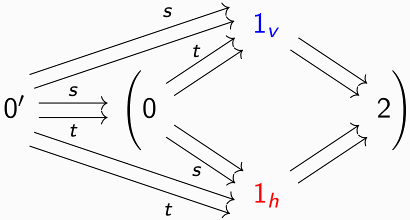


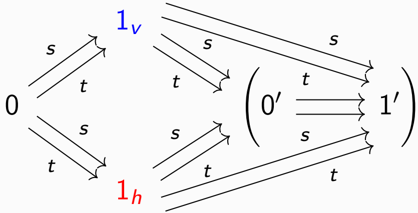


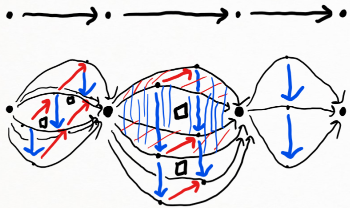




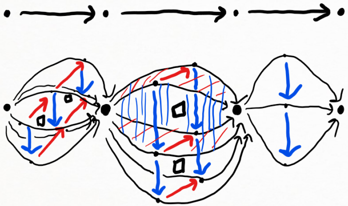
- Cell shapes of $\mathcal{C} \wr_e \mathcal{D}$ are e-cells stuffed with cell shapes of $\mathcal{D}$
- Cell diagrams in $\widehat{\mathcal{C} \wr_e \mathcal{D}}$ are cell diagrams over $\mathcal{C}$ stuffed with a diagram over $\mathcal{D}$ in each e-cell





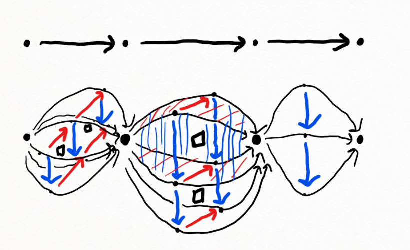


Enrichment via Cell Shapes

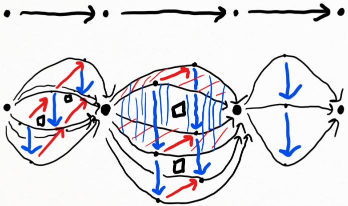


Enrichment via Cell Shapes

- Let $T_{\mathcal{C}}$, $T_{\mathcal{D}}$ be familial monads on $\widehat{\mathcal{C}}$, $\widehat{\mathcal{D}}$

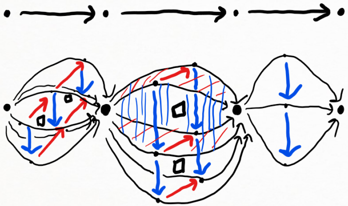


- Let $T_{\mathcal{C}}$, $T_{\mathcal{D}}$ be familial monads on $\widehat{\mathcal{C}}$, $\widehat{\mathcal{D}}$
- Build composable diagrams over $\mathcal{C} \wr_e \mathcal{D}$ by stuffing those over $\mathcal{C}$ with composable diagrams over $\mathcal{D}$



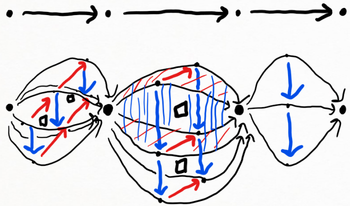
Enrichment via Cell Shapes

- Let $T_{\mathcal{C}}$, $T_{\mathcal{D}}$ be familial monads on $\widehat{\mathcal{C}}$, $\widehat{\mathcal{D}}$
- Build composable diagrams over $\mathcal{C} \wr_e \mathcal{D}$ by stuffing those over $\mathcal{C}$ with composable diagrams over $\mathcal{D}$
- (S.) These diagrams represent a familial monad T on $\widehat{\mathcal{C} \wr_e \mathcal{D}}$.



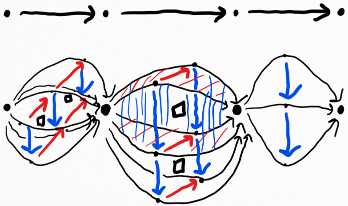
Enrichment via Cell Shapes

- Let $T_{\mathcal{C}}$, $T_{\mathcal{D}}$ be familial monads on $\widehat{\mathcal{C}}$, $\widehat{\mathcal{D}}$
- Build composable diagrams over $\mathcal{C} \wr_e \mathcal{D}$ by stuffing those over $\mathcal{C}$ with composable diagrams over $\mathcal{D}$
- (S.) These diagrams represent a familial monad T on $\widehat{\mathcal{C} \wr_e \mathcal{D}}$.
- (S.) When $\mathcal{C} = G_1$, T-algebras $\simeq$ $T_{\mathcal{D}}$-enriched categories.



Enrichment via Cell Shapes

- Let $T_{\mathcal{C}}$, $T_{\mathcal{D}}$ be familial monads on $\widehat{\mathcal{C}}$, $\widehat{\mathcal{D}}$
- Build composable diagrams over $\mathcal{C} \wr_e \mathcal{D}$ by stuffing those over $\mathcal{C}$ with composable diagrams over $\mathcal{D}$
- (S.) These diagrams represent a familial monad T on $\widehat{\mathcal{C} \wr_e \mathcal{D}}$.
- (S.) When $\mathcal{C} = G_1$, T-algebras $\simeq$ $T_{\mathcal{D}}$-enriched categories.
- (S.) When $T_{\mathcal{C}}$ is "e-injective" and $T_{\mathcal{D}}$ "has enough degeneracies", the theory $(\mathcal{C} \wr_e \mathcal{D})_T \simeq \mathcal{C}_{T_{\mathcal{C}}} \wr \mathcal{D}_{T_{\mathcal{D}}}$ where $\mathcal{C}_{T_{\mathcal{C}}} \to \Gamma$ counts the e-cells in each $E_{\mathcal{C}} t$.



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- Mark Weber, *Familial 2-Functors and Parametric Right Adjoints*, Theory and Applications of Categories, Vol. 18, No. 22, 2007, pp. 665–732.

Thank You!