Dynamic Operads for Evolving Organizations

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TOPOS INSTITUTE

Outline

1. Nested dynamic organizations
2. Organizations: polynomials and wiring diagrams
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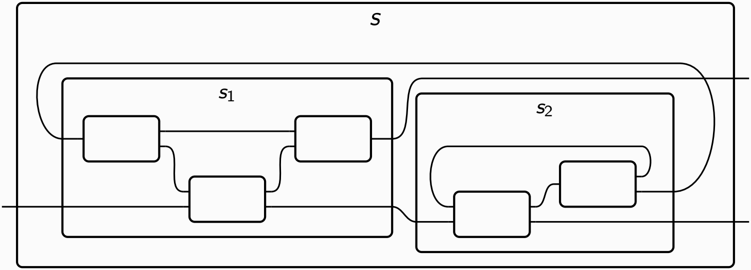
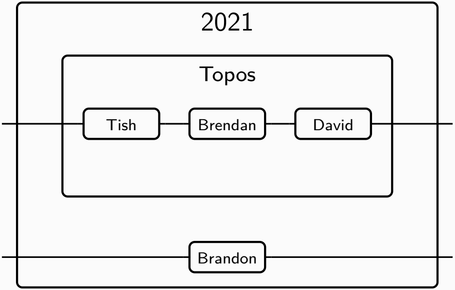
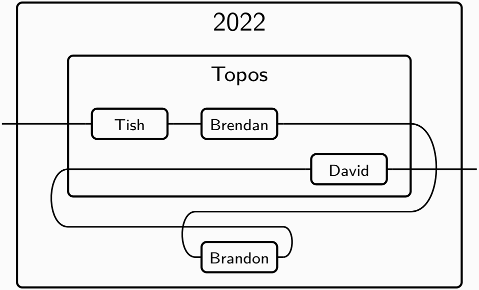
5. A dynamic weighted prediction market

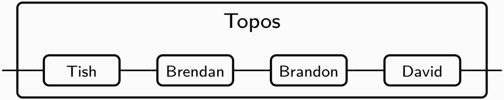


Nested dynamic organizations

- How I joined Topos Institute





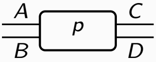


(Not an accurate representation of Topos Institute's internal structure)

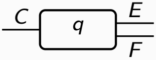
Organizations: polynomials and wiring diagrams

- Let A, B, C, D, E, F, G be sets, and consider the polynomials

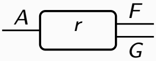
$$p = CDy^{AB}$$



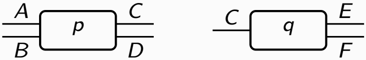
$$q = EFy^{C}$$



$$r = FGy^{A}$$



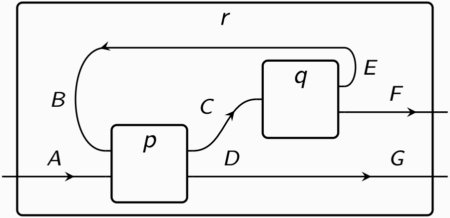
$$p \otimes q = CDEFy^{ABC}$$



- Polynomials form a category where a morphism $p \otimes q \to r$ consists of functions

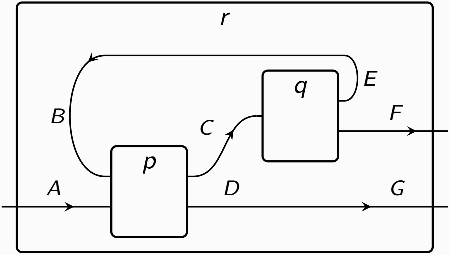
$$CDEF \to FG$$

$$(CDEF)A \to ABC$$



Dynamics: polynomial coalgebras

- A p-coalgebra is a set S of "states" with a function $S \to p(S)$
- For $p(S) = CD \times S^{AB}$, each state is assigned an element of CD and a function $AB \to S$ which updates the state
- $[p \otimes q, r]$ is the polynomial $\mathrm{Hom}_{\mathbf{Poly}}(p \otimes q, r) \times y^{ACDEF}$
- A $[p \otimes q, r]$-coalgebra consists of, for each state $s \in S$, a "wiring" $p \otimes q \to r$ and a "rewiring" function $ACDEF \to S$



Nesting: operad structure

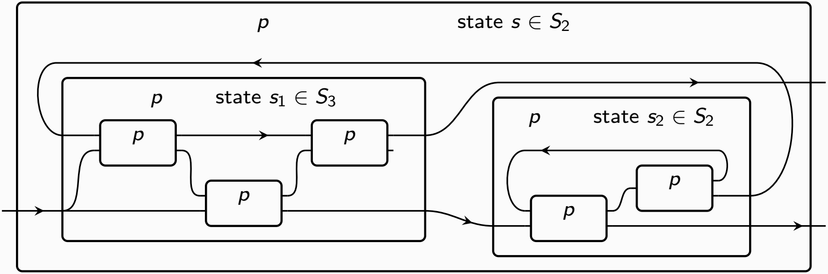
- An operad S consists of sets S_n of n-ary operations for all $n \in \mathbb{N}$ with compatible unit and composition operations

$$1 \xrightarrow{\eta} S_1, \qquad S_n \times S_{m_1} \times \cdots \times S_{m_n} \xrightarrow{\mu} S_{m_1 + \cdots + m_n}$$

- A *dynamic operad* on p is an operad S along with coalgebras

$$S_n \to [p^{\otimes n}, p](S_n)$$

for all n, such that η and μ respect wirings and rewirings



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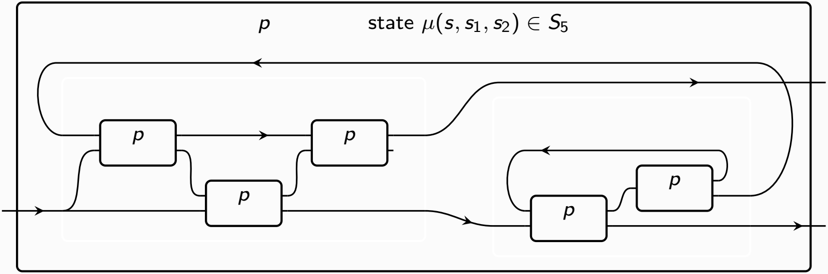
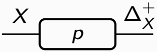


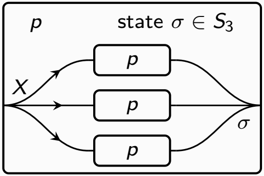
A dynamic weighted prediction market

- Let Δ_X^+ be the set of nowhere-zero probability distributions on a finite set X
- Let $p = \Delta_X^+ y^X$, where $p^{\otimes n} = (\Delta_X^+)^n y^{X^n}$



- Let $S_n = \Delta_{\underline{n}}^+$, for $\underline{n}$ the set with n elements (players), with an operad structure given by convex combination
- The wiring of a state $\sigma = (\sigma_1, ..., \sigma_n) \in S_n$ sends $\tau^1, ..., \tau^n$ to $\sigma_1\tau^1 + \cdots + \sigma_n\tau^n$ and x to $(x, ..., x)$
- The rewiring $(\Delta_X^+)^n \times X \to \Delta_n^+$ sends $\tau^1, ..., \tau^n, x$ to σ' where

$$\sigma'_i = \frac{\sigma_i \tau_x^i}{\sum_j \sigma_j \tau_x^j}$$



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- Brandon T. Shapiro and David I. Spivak, "Dynamic operads, dynamic categories: From deep learning to prediction markets" arXiv:2205.03906
- Matteo Capucci, Riu Rodriguez Sakamoto, Brandon T. Shapiro, and David I. Spivak, "A dynamic monoidal category for strategic games" Topos Institute Blog
- Sophie Libkind and David I. Spivak, "When you light up, I light up: A dynamical monoidal category of Hebbian learners" Topos Institute Blog

Thanks!