Dynamic Operads for Evolving Organizations

Brandon Shapiro* and David Spivak

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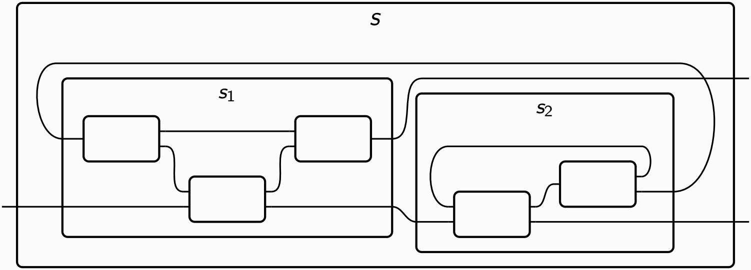
Outline

1. Motivation: dynamic organization with abstractions
2. Morphisms of polynomials are wiring diagrams
3. Polynomial coalgebras describe dynamics
4. Operad structure encodes nested abstraction
5. A dynamic weighted prediction market



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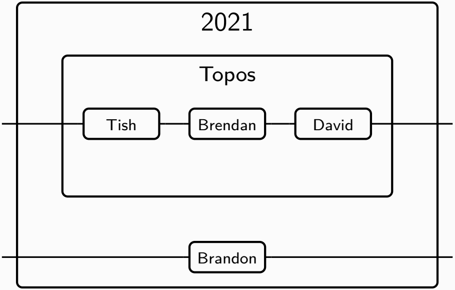

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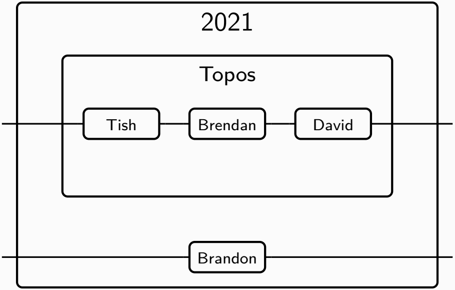

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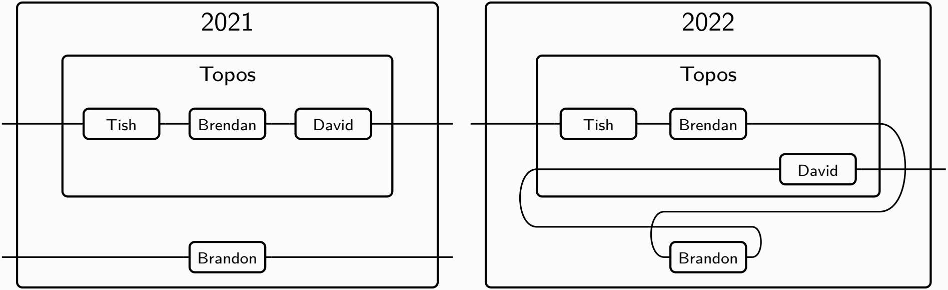

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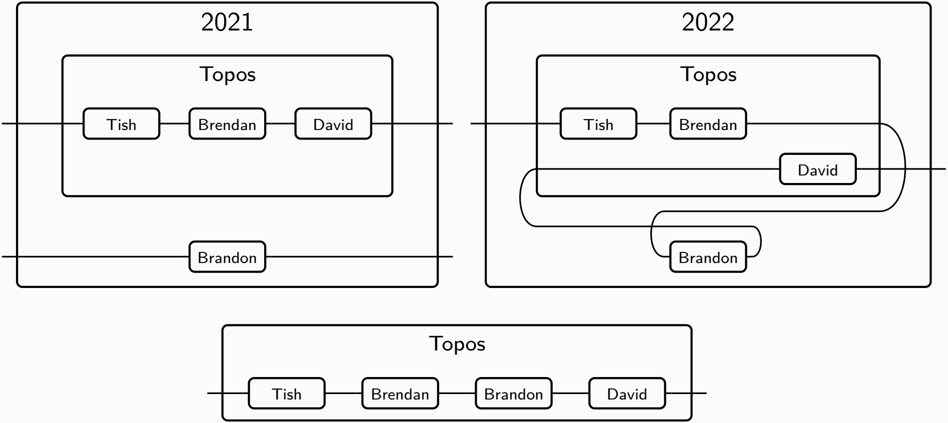

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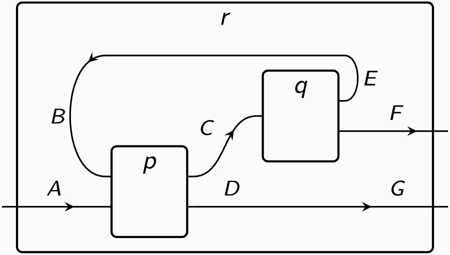
- For functions $D \to G$ and $E \to B$, one example is depicted by



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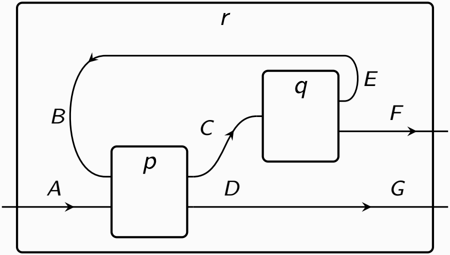

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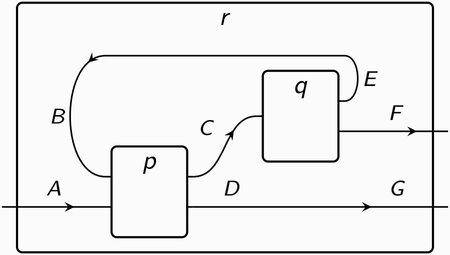

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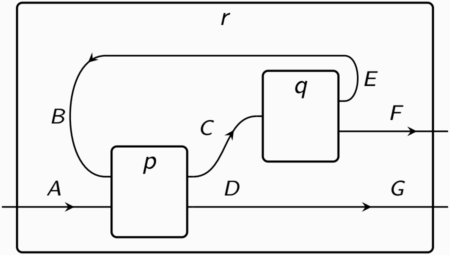
- $[p \otimes q, r] = \mathrm{Hom}_{\mathbf{Poly}}(p \otimes q, r) \times y^{ACDEF}$
- A $[p \otimes q, r]$-coalgebra consists of, for each state $s \in S$, an "action" $\phi_s : p \otimes q \to r$ and an "update" $ACDEF \to S$



Operads=nested abstraction

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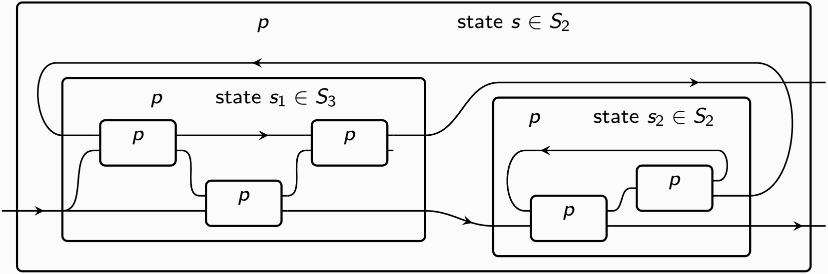

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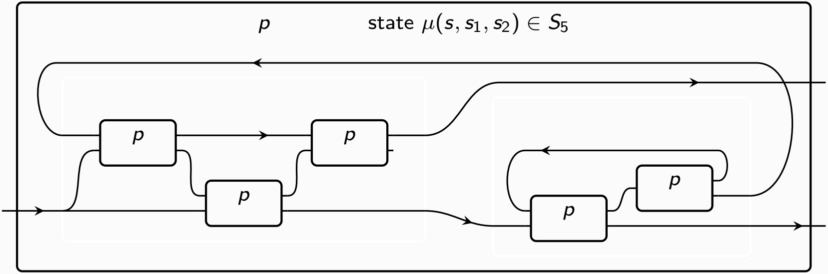

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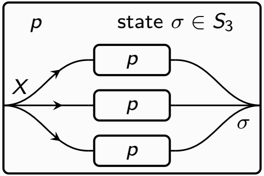

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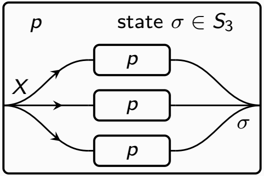
- A state $\sigma = (\sigma_1, \ldots, \sigma_n) \in S_n$ has action as below and update $(\Delta_X^+)^n \times X \to \Delta_n^+$ sending $\tau^1, \ldots, \tau^n, x$ to σ' where

$$\sigma'_i = \frac{\sigma_i \tau_x^i}{\sum_j \sigma_j \tau_x^j}$$



References

- Brandon T. Shapiro and David I. Spivak, "Dynamic categories, dynamic operads: From deep learning to prediction markets" arXiv:2205.03906

Thanks!