Dynamic Operads for Evolving Organizations

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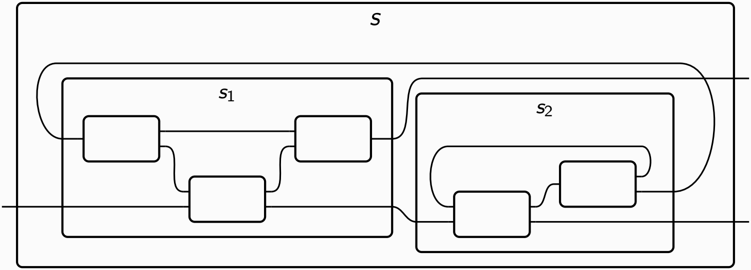
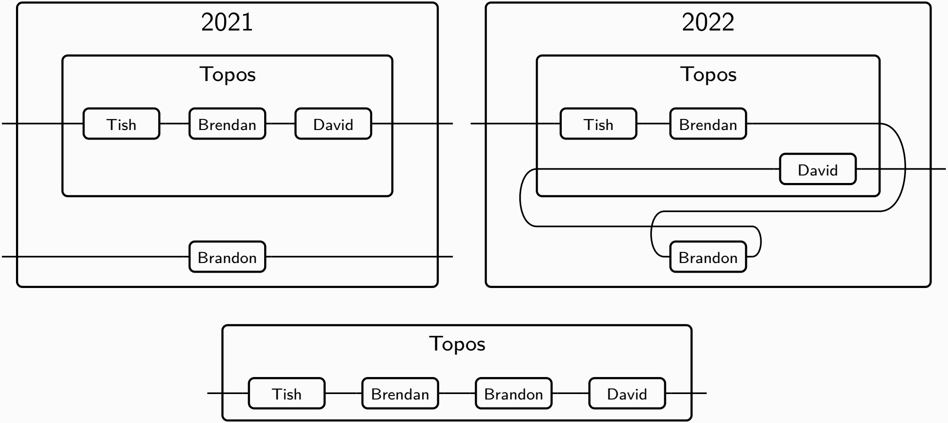
Outline

1. Motivation: dynamic organization with abstractions
2. Morphisms of polynomials are wiring diagrams
3. Polynomial coalgebras describe dynamics
4. Operad structure encodes nested abstraction
5. A dynamic weighted prediction market



Organizational change

- How I joined Topos Institute



(Not an accurate representation of Topos Institute's internal structure)

Morphisms of polynomials = wiring diagrams

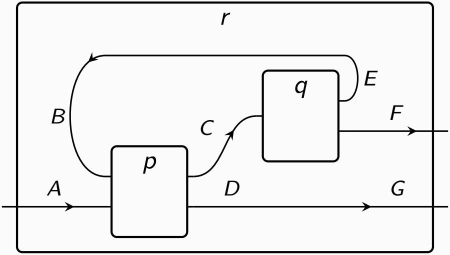
- Let A, B, C, D, E, F, G be sets, and consider the polynomials

$$p = CDy^{AB}, \qquad q = EFy^{C}, \qquad r = FGy^{A}, \qquad p \otimes q = CDEFy^{ABC}$$

- A morphism $p \otimes q \to r$ consists of functions

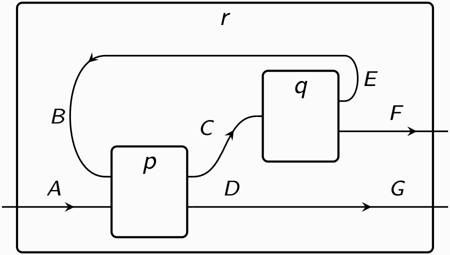
$$CDEF \to FG, \qquad ACDEF \to ABC$$

- For functions $D \to G$ and $E \to B$, one example is depicted by



Coalgebras as dynamics

- A p-coalgebra is a set S of "states" with a function $S \to p(S)$
- $[p \otimes q, r] = \text{Hom}_{\textbf{Poly}}(p \otimes q, r) \times y^{ACDEF}$
- A $[p \otimes q, r]$-coalgebra consists of, for each state $s \in S$, an "action" $\phi_s : p \otimes q \to r$ and an "update" $ACDEF \to S$



Operads=nested abstraction

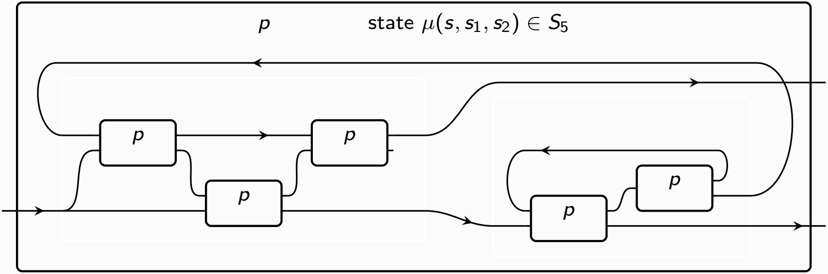
- An operad S consists of sets S_n of n-ary operations for all $n \in \mathbb{N}$ with unit and composition

$$1 \xrightarrow{\eta} S_1, \qquad S_n \times S_{m_1} \times \cdots \times S_{m_n} \xrightarrow{\mu} S_{m_1+\cdots+m_n}$$

- A *dynamic operad* on p is an operad S along with coalgebras

$$S_n \to [p^{\otimes n}, p](S_n)$$

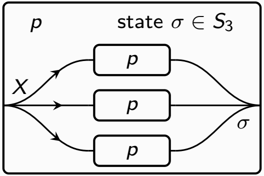
for all n, such that η and μ respect actions and updates



Dynamic prediction making

- Let Δ_X^+ be the set of nowhere-zero probability distributions on a finite set X
- Let $p = \Delta_X^+ y^X$, where $p^{\otimes n} = (\Delta_X^+)^n y^{X^n}$
- Let $S_n = \Delta_{\underline{n}}^+$, for $\underline{n}$ the set with n elements (players), with composition given by convex combination
- A state $\sigma = (\sigma_1, \ldots, \sigma_n) \in S_n$ has action as below and update $(\Delta_X^+)^n \times X \to \Delta_n^+$ sending $\tau^1, \ldots, \tau^n, x$ to σ' where

$$\sigma_i' = \frac{\sigma_i \tau_x^i}{\sum_j \sigma_j \tau_x^j}$$



References

- Brandon T. Shapiro and David I. Spivak, "Dynamic categories, dynamic operads: From deep learning to prediction markets" arXiv:2205.03906

Thanks!